





The one property of a parabola that will help us find specific values along the parabola and its focus is that all parabolas are similar.

Theory:

Any triangle can be "summed" by geometrically dividing it into 2 or more right triangles.

Worked example of "Trigonometric Parabola". With diagrams.

We are looking to find what angles of a triangle make up a specifically designed parabola. This is an application of the "Trigonometric Parabola". Do not confuse the steps to find a pattern among Prime numbers on a parabola with the right triangle work. Again, we solve the right triangles to test our parabola.

Given:

We have an unknown value that the focus occurs. And we have a triangle that could be of infinite combinations. We only know that the lengths of 2 of the sides. The smaller length is given as 3 (the first Prime number) and the longer length is 5 (the second Prime number).

Find:

The triangle that describes an angle in this example that finds all our needed values.

Solution:

First we must split our unknown angle into two right triangles.

Now if we can solve those triangles. In this example it is easiest to solve for the two right triangles having a hypotenuse of 3 and a hypotenuse of 5 respectively.

Given:

$$(\cos x)^2 + (\sin y)^2 = 1$$

$$\sin\theta = \frac{\textit{Opposite}}{\textit{Hypotenuse}} \textit{ on a right triangle}$$

$$\cos\theta = \frac{\textit{Adjacent}}{\textit{Hypotenuse}} \textit{ on a right triangle}$$

$$\textit{tangent}\theta = \frac{\textit{Opposite}}{\textit{Adjacent}} \textit{ on a right triangle}$$

$$\textit{The Pythagorean Theorem } a^2 + b^2 = c^2$$

and the ability to substitute variables in an equation

For a hypotenuse of 3:

$$L = \textit{Scosine} = \frac{|\cos[\theta_1] - \cos\theta_2|}{\cos[\theta_2]} \cdot r$$

$$\cos[\theta_1] = 1$$

3 our given Prime number which is also the hypotenuse becomes

$$3 = \frac{1}{L} + x$$

The cosine of the right triangle is $\frac{x}{3}$

$$3 = \frac{1}{\frac{1-x}{3}} + y$$

$$\frac{\frac{x}{3}}{1 - \frac{x}{3}} + y = 3$$

$$\frac{\frac{x}{3}}{1 - \frac{x}{3}} = 3 - y$$

$$\frac{x}{3} = (1 - \frac{x}{3})(3 - y)$$

$$x = \frac{1}{3} (3 - y - x + \frac{x \cdot y}{3})$$

$$x = 1 - \frac{y}{3} - \frac{x}{3} + \frac{xy}{9}$$

$$3 \cdot (\frac{4}{3} \cdot x - 1) = -y + \frac{xy}{3}$$

$$4x - 1 = -y + \frac{xy}{3}$$

$$\frac{3 \cdot (4x - 1)}{x \cdot y} = -y$$

$$\frac{3 \cdot (4x - 1)}{x} = -y^2$$

$$y^2 = -1 \cdot \left[\frac{3 \cdot (4x - 1)}{x} \right]$$

as previously derived

$$y = \sqrt{(3 - x)(3 + x)}$$

$$y = \sqrt{3^2 - x^2}$$

substitute

$$[\sqrt{3^2 - x^2}]^2 = -1 \cdot \left[\frac{3 \cdot (4x - 1)}{x} \right]$$

$$x^2 - 3^2 = \frac{3 \cdot (4x - 1)}{x}$$

$$x^3 - 9x = 3 \cdot (4x - 1)$$

$$3x^3 - 27x = 4x - 1$$

$$3x^3 - 27x + 1 = 4x$$

$$3x^2 + \frac{1}{x} - 27 = 4$$

$$3x^2 + \frac{1}{x} - 23 = 0$$

Plug into the quadratic equation:

$$\frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$$

$$\frac{-1 \pm \sqrt{-1^2 - 4 \cdot 3 \cdot -23}}{2 \cdot 3}$$

$$\frac{-1 \pm \sqrt{1 + 276}}{6}$$

$$\frac{-1 + \sqrt{277}}{6} \text{ or } \frac{-1 - \sqrt{277}}{6}$$

$$x = 2.6072 \text{ or } x = -2.9405$$

Plug into right triangle trigonometric ratios:

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \text{ on a right triangle}$$

$$\cos\theta = \frac{2.6072}{3}$$

$$\theta = \cos^{-1}\left[\frac{2.6072}{3}\right]$$

$$\theta = 29.6496 \text{ degrees}$$

So we basically have a 30---60 triangle and can use this info to find where an arc of π radians intersects a parabola with the distance from the apex equal to our Prime number 3!

And for a hypotenuse of 5:

$$L = \text{Scosine} = \frac{|\cos[\theta_1] - \cos\theta_2|}{\cos[\theta_2]} \cdot r$$

$$\cos[\theta_1] = 1$$

3 our given Prime number which is also the hypotenuse becomes

$$5 = \frac{1}{L} + x$$

The cosine of the right triangle is $\frac{x}{5}$

$$5 = \frac{1}{\frac{1-x}{\frac{x}{5}}} + y$$

$$\frac{\frac{x}{5}}{1-\frac{x}{5}} + y = 5$$

$$\frac{\frac{x}{5}}{1-\frac{x}{5}} = 5 - y$$

$$\frac{x}{5} = (1 - \frac{x}{5})(5 - y)$$

$$x = \frac{1}{5}(5 - y - x + \frac{x \cdot y}{5})$$

$$x = 1 - \frac{y}{5} - \frac{x}{5} + \frac{xy}{25}$$

$$5 \cdot (\frac{4}{5} \cdot x - 1) = -y + \frac{xy}{5}$$

$$4x - 1 = -y + \frac{xy}{5}$$

$$\frac{5 \cdot (4x - 1)}{x \cdot y} = -y$$

$$\frac{5 \cdot (4x - 1)}{x} = -y^2$$

$$y^2 = -1 \cdot [\frac{5 \cdot (4x - 1)}{x}]$$

as previously derived

$$y = \sqrt{(5 - x)(5 + x)}$$

$$y = \sqrt{5^2 - x^2}$$

substitute

$$[\sqrt{5^2 - x^2}]^2 = -1 \cdot [\frac{5 \cdot (4x - 1)}{x}]$$

$$x^2 - 5^2 = \frac{5 \cdot (4x - 1)}{x}$$

$$x^3 - 25x = 5 \cdot (4x - 1)$$

$$5x^3 - 125x = 4x - 1$$

$$5x^3 - 125x + 1 = 4x$$

$$5x^2 + \frac{1}{x} - 125 = 4$$

$$5x^2 + \frac{1}{x} - 125 = 0$$

Plug into the quadratic equation:

$$\frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$$

$$\frac{-1 \pm \sqrt{-1^2 - 4 \cdot 5 \cdot -125}}{2 \cdot 5}$$

$$\frac{-1 \pm \sqrt{1+500}}{10}$$

$$\frac{-1 + \sqrt{501}}{10} \text{ or } \frac{-1 - \sqrt{501}}{10}$$

$$x = 2.1383029 \text{ or } x = -2.33830$$

Plug into right triangle trigonometric ratios:

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \text{ on a right triangle}$$

$$\cos\theta = \frac{2.1383029}{5}$$

$$\theta = \cos^{-1}\left[\frac{2.1383029}{5}\right]$$

$$\theta = 64.6808 \text{ degrees}$$

A triangle that is split into 2 separate right triangles with a hypotenuse of 5 will have a corresponding angle of 64 degrees.

If this works it means we can split any triangle into 2 right triangles and solve for the triangle knowing only 2 sides!!!!!!

The only part in question is $b = 1$.

$$5x^2 + 1x^{-1} - 125 = 0$$

Solve the unknown sides:

$$[\sin(64.6808 \text{ degrees}) \cdot 5] - [\sin(29.6496 \text{ degrees}) \cdot 3] = e$$

$$[\cos(64.6808 \text{ degrees}) \cdot 5] - [\cos(29.6496 \text{ degrees}) \cdot 3] = d$$

$$d^2 + e^2 = f^2$$

$$f = \text{side } b$$

Now 3 sides and an angle are known!