

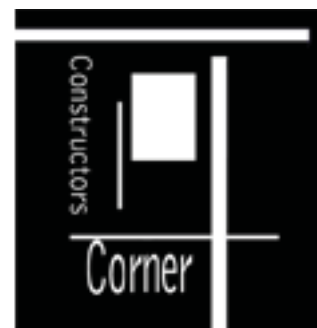
Contents

**Constructor's
Corner**

It's all about

IDEAS

www.constructorscorner.com



Page 00001--- Arched Door
Page 00017--- SCos SSine
Page 00027--- Unknown Curves
Page 00035--- Extraordinary Ellipse
Page 00043--- Cell Tower
Page 00047--- Mixture Problems
Page 00055--- Billard Ball
Page 00065--- Square Circle
Page 00071--- Circle Key

Page 00081--- Linear Parabola
Page 00091--- Chord Inverse
Page 00093--- Chord Application
Page 00097--- Chord vs Circular Function
Page 00101--- Invalute vs Logarithmic Spiral
Page 00107--- Circle-Parabola-Proportions
Page 00113--- Parabola vs Exponent
Page 00121--- Video Game Curves
Page 00125--- Sonar Coordinates

Page 00129--- Instructor's Arched Door
Page 00133--- Baby Seat - Dynamics

This is a rough draft of a collection of math and design problems. It may be printed out and distributed as long as credit is given to Constructor's Corner and the web address "www.constructorscorner.com" is also printed.

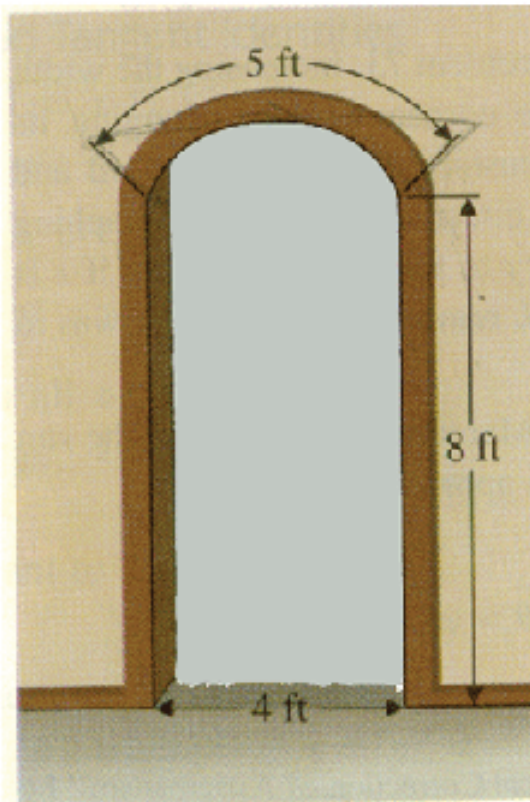
Being a draft and edited rapidly there are many mistakes both in the math and the syntax. If you find a mistake in the math logic or the grammar of the writeup, email me at trurlthe_constructor@hotmail.com

Arched Door

Intro: this is a math problem I have been working on for some time. It is quite possibly an error in the text. I will tell you from the start neither the angle of the circle or the radius is given. Perhaps you might discover a new way to define circles or arcs when trying to solve this problem. It can be solved using higher mathematics but I want to find a simpler method that is simple yet brilliant in its design.

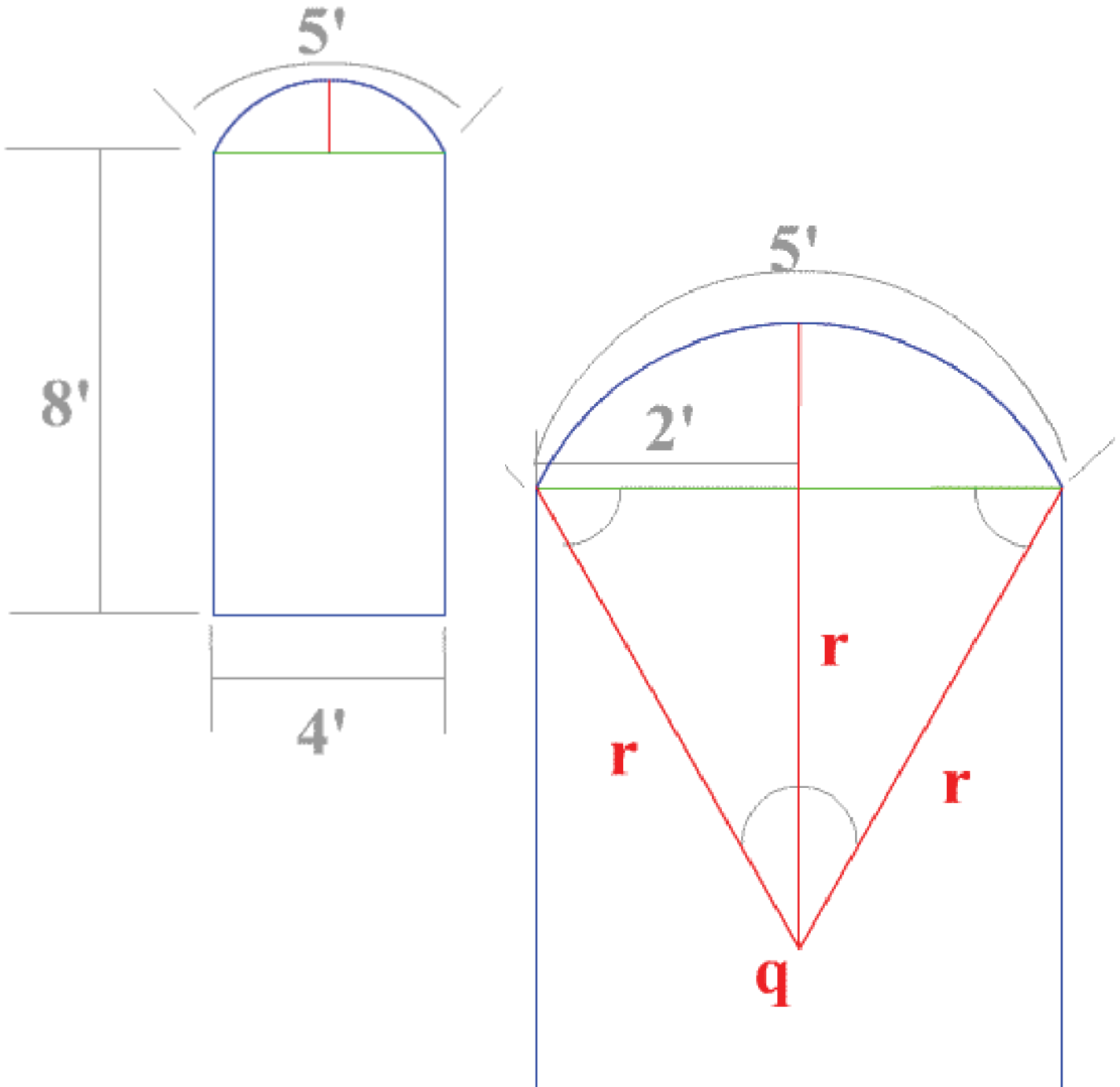
I am also going to post more pages that go together with this problem in the future. But I just want to see how you approach the problem before I reveal my strategies. And together maybe we can find a extraordinary answer.

- c 71. Architecture** An arched doorway is formed by placing a circular arc on top of a rectangle (see the figure). If the rectangle is 4 ft wide and 8 ft high and the circular arc is 5 ft long, what is the area of the doorway? Compute answer to two decimal places.



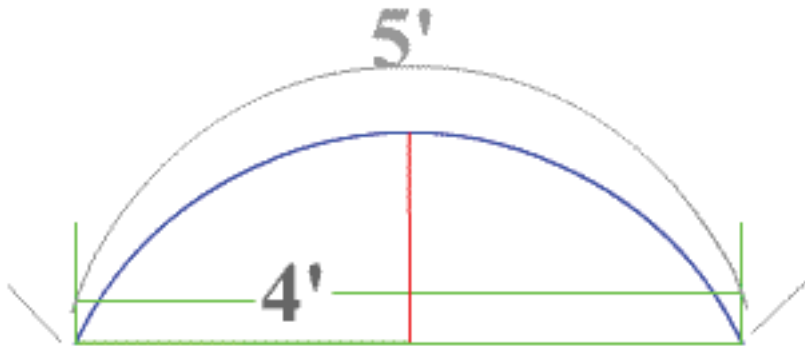
this problem is excerpted from:
page 281 problem 71 of:
**Analytic Trigonometry with
Applications Sixth Edition**
Authors: Barnett and Ziegler

Figure for 71



Here is another look at what is given in the problem. I think we should concentrate on these 3 radiuses. Point “q” is the place where they met. So we have an angle whose sin equals 4'. And whose creates an arc whose length equals 5'.

I have tried to work on a new way to solve for these values. But every method I try you have to find the value of the angle at “q.” This is why I invented the Scos and Ssin to work backwards and find the angle and radius. But every attempt has failed because I still need to know the radius or the angle. So if anyone can find the angle this problem is solved.



This is a new look at the same arched door. First we will find the chord that is in the center of the doorway.

But know it is important to look at the given and find those clues that we may not notice at first glance.

Now direct your attention to known lengths that we have solved underneath the arc.

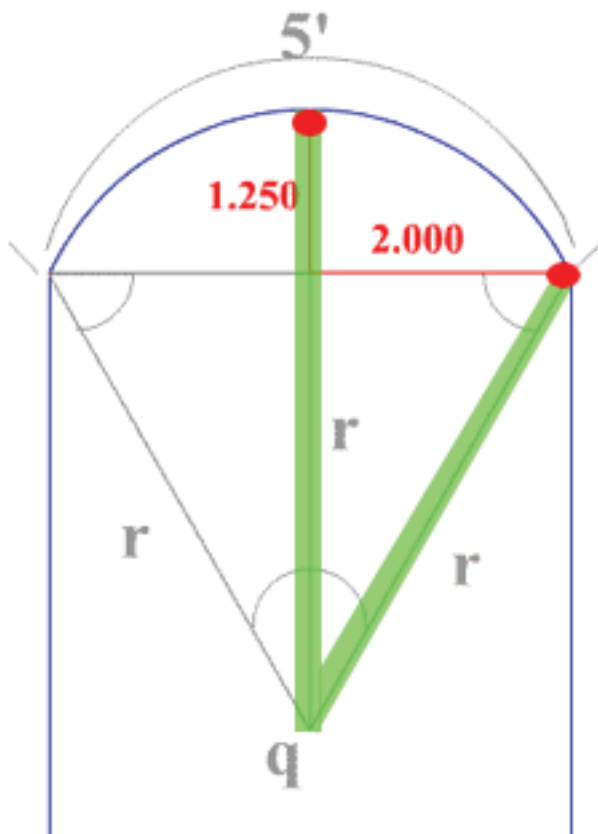
Refer to the next picture below.

Since the arc is 5' and the linear distance covered by the arc is 4',

the distace of the **chord** in the center of the arc is given by the equation:

$$\text{Chord} = \text{Length of arc} / \text{Length of x distance}$$

$$\text{so Chord} = 5/4 = 1.250'$$



Now we have sides of the right triangle. But this is not the angle that we are concerned with.

Look at the radius that is at $\pi/2$ and the radius that is at the right of it that is at an unknown angle we must find.

Here is what we know:

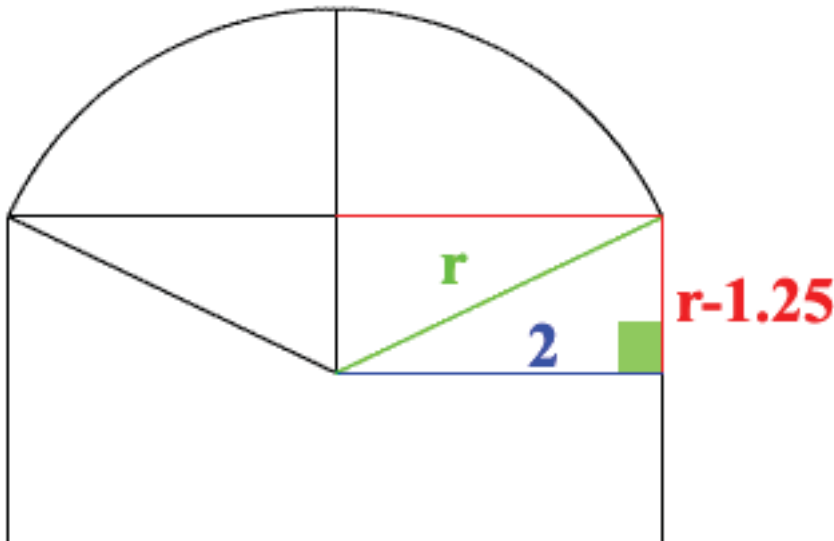
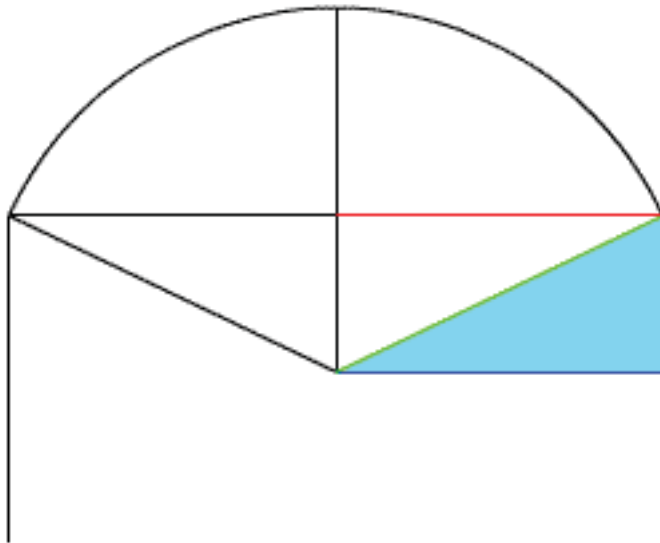
As the cos of the angle (between the two radius) moves 2 feet it makes an arc of 2.5 feet. At this time the sin of this angle with radius r moves 1.25 feet.

So the two lengths that we have found in this picture show how much the sine and cosine have change between these two radius.

Let us look at the drawing drawn to scale. We can see the triangle (highlighted in blue) that is formed by the values we just found from the chord and the given 2 foot x segment.

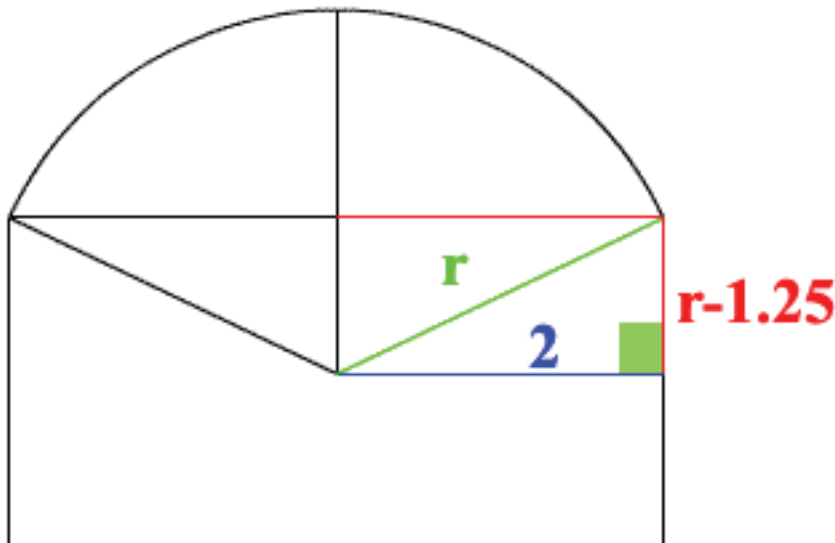
Try to picture in your mind what makes this triangle and the value of its sides so special.

As you can see in the next example these values can be used in the Pythagorean Theorem to solve for r :) !!!!!!!



Here are the values. The value of 2 is the hardest if possible to find. Luckily it was given in the problem by $4/2$.

See if you can now solve for r .



By the Pythagorean Theorem :

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 (1.25)^2 + 2^2 &= r^2 \\
 r^2 - 1.25r - 1.25r + 1.5625 + 4 &= r^2 \\
 r^2 - 2.5r + 5.5625 &= r^2 \\
 -2.5r &= -5.5628 \\
 r &= 2.22512
 \end{aligned}$$

By the equation :

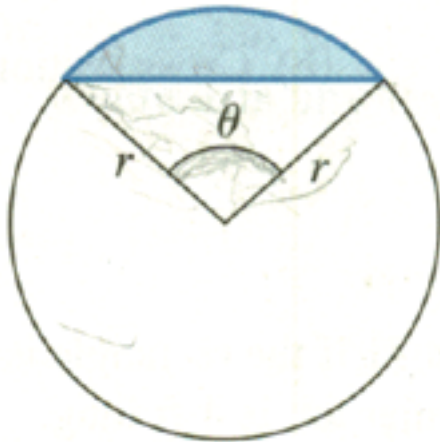
$$\begin{aligned}
 \theta &= s/R \\
 \theta &= 2.5/2.22512 \\
 \theta &= 1.12534990477 \\
 \theta &= 1.1253 \text{ radians}
 \end{aligned}$$

Check by the definition of cosine :

$$\begin{aligned}
 \cos((\pi/2) - \theta) * r &= 2 \\
 \cos(0.4454 \text{ radians}) * 2.22512 &= 2 \\
 2 &= 2 \\
 \text{It checks!!!}
 \end{aligned}$$

c 69. Geometry The area of a segment of a circle in the figure is given by

$$A = \frac{1}{2} r^2 (\theta - \sin \theta)$$

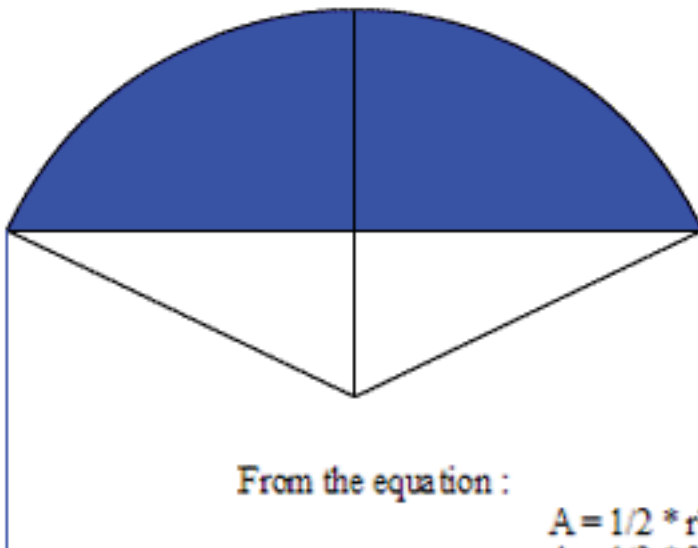


This formula is from page 281
"Analytical Trigonometry with Applications"
Barnett and Ziegler

This is the formula used to
determine the area of the
equation.

Figure for 69

Follow the next steps.



From the equation :

$$A = \frac{1}{2} * r^2 (\theta - \sin (\theta))$$

$$A = \frac{1}{2} * 2.22512 * (((1.1253 \text{rad} * 2) - \sin((1.1253 * 2))))$$

$$A = 2.47557 * 1.472903$$

$$A = 3.64288$$

$$A = 3.65$$

The total area of the door is as follows:

$$\text{Length} * \text{Width} = \text{Height}$$

$$8 * 4 = 32$$

$$\text{Total area} = \text{area of arc} + \text{area of rectangle}$$

$$\text{The area of the door} = 32 + 3.65 = 35.65$$

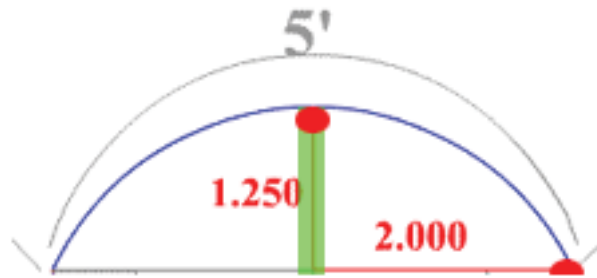
Is the problem now totally solved using basic mathematics???

There is a lot to learn from this problem. The steps to this problem could possibly be used to solve other problems.

But the thing to study here is the relationship on how a change in the cosine or a change in the sine form a triangle. This problem has other solutions. But higher mathematics must be used to solve them.

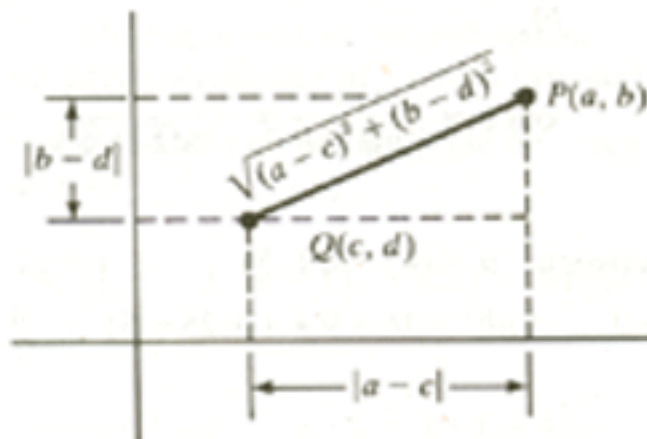
Remember if you have any questions or comments just leave them on the message board.

If we take a look at how the arc doorway problem was solved by just using elementary mathematics (below calculus). Basically we are solving for two lengths. One is the width of the height of the arc in the center from the base at each end. And the other length is the length between the two arc ends.



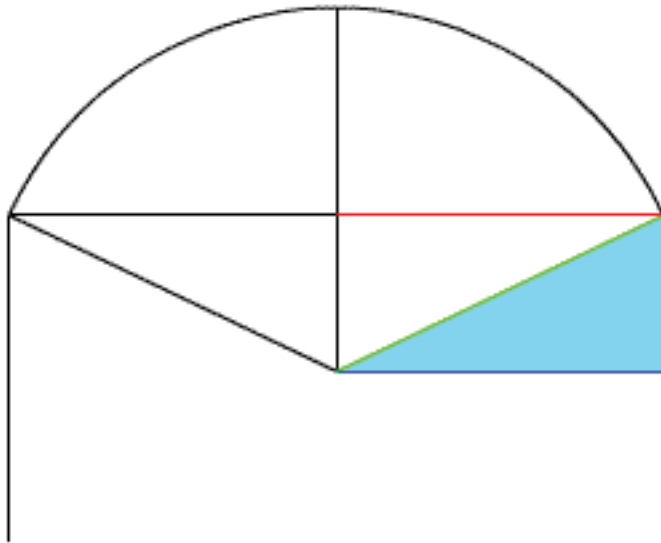
But the fact that all three segments are all radii that meet at the same center point we are able to determine variables for the sides. Since the middle length is known (1.25), we know that the length of one of the sides of the right triangle is $r - \text{height of arc from arc ends or } r - 1.25$ in this problem. And the length from the end of the arc to the bottom center of the arc is given as 2 in this problem. We then know the third side is one of the radii and is the unknown in this equation. Thus we can use the Pythagorean theorem we can solve for the equation:

$$a^2 - b^2 = c^2 \text{ or } (r - 1.25)^2 - 2^2 = c^2$$



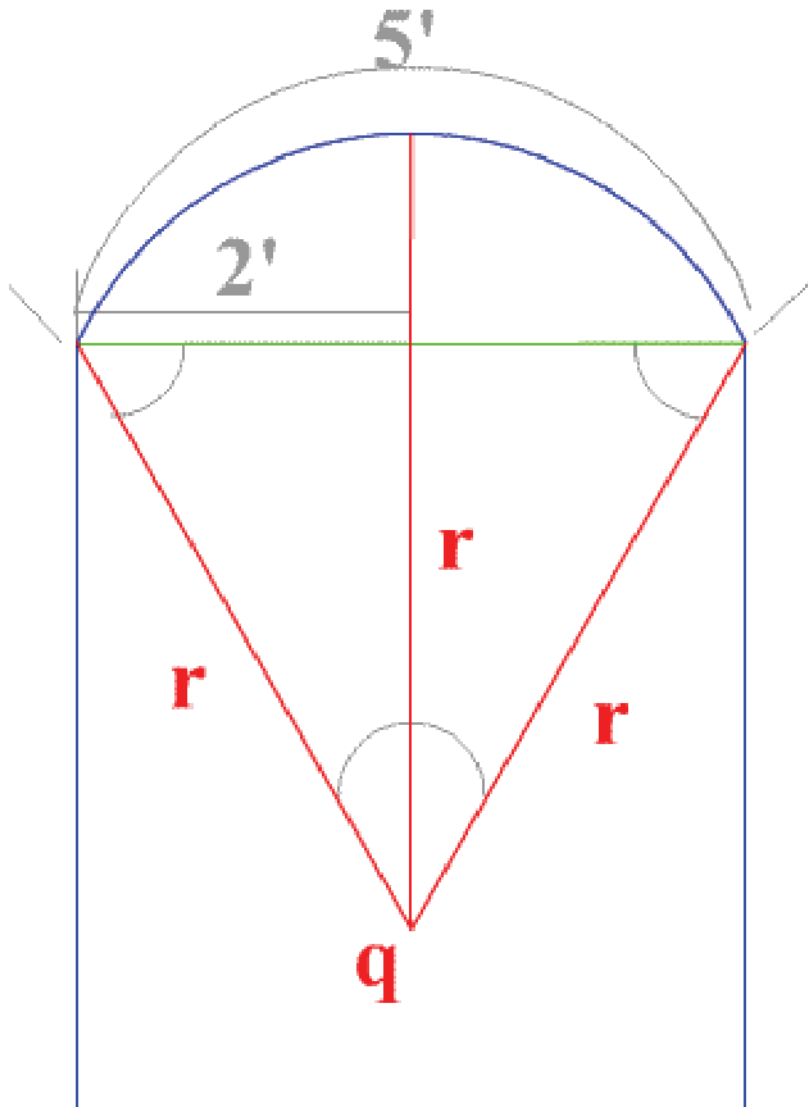
**Here $b - d = r - 1.25$
and
 $a - c = 2$**

diagram from
Schaum's Solved Problems
3000 solved problems in
Linear Algebra



What have we just solved here? It is so simple once you see it. The solution uses the Pythagorean and a form of simple addition and subtraction along the coordinate plane.

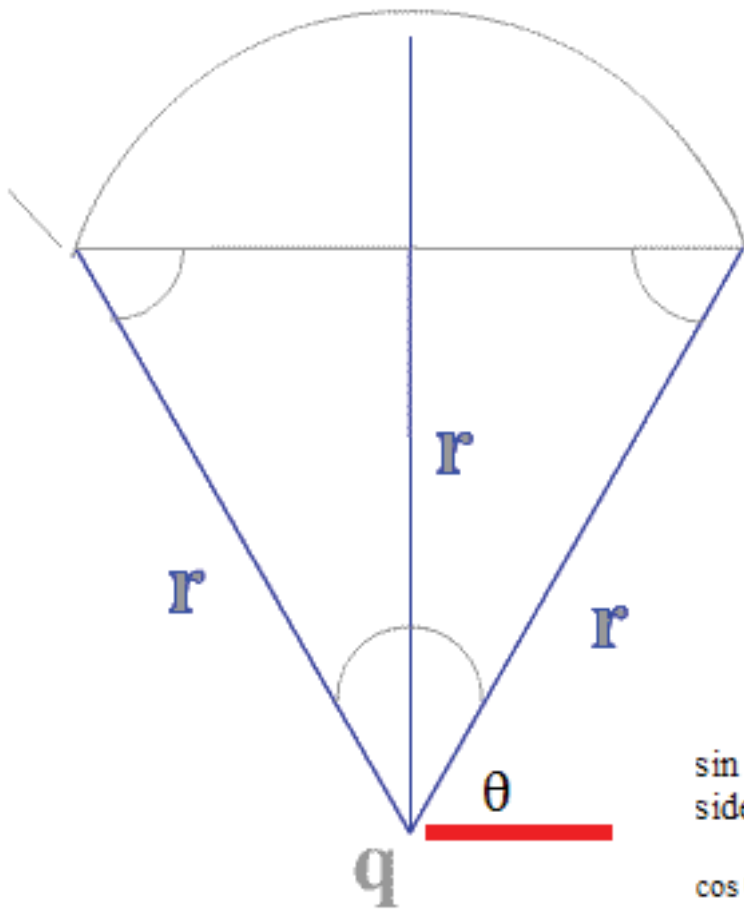
But what about those three radius. Is there anything more to discover with them. We took information about those three radiuses without knowing their values and used the two lengths of the arc to find the value of the radiuses. And once you know the radius you can find many things about a circle. But those three radiuses not only find the value of the radius it determines the whole arc.



We can describe the circular arc using just three radiuses. Since the arc is symmetric in length and shape (and it is a circular arc) it can be described by use of an isosceles triangle! The sides being of equal length and a base that equal the radius minus the height of the center arc length. And base plus the height of the center arc is equal to the sides (other radiuses) of the isosceles triangle.

Here are three radius in an isosceles triangle.
These three lengths determine the size and shape of the circular arc above.

Here the two sides equal to the radius of the arc and the base plus the segment equals the radius so:



$$\sin \theta * \text{side} = \text{base}$$

$$\text{side} = \text{base} + \text{segment} = r$$

$$\cos \theta * \text{side} = \frac{1}{2} \text{ the distance between the arc ends}$$

$$= 2 \text{ (in arc door problem) = the bottom center of arc to the one end of the arc}$$

A circle is really made up of many isosceles triangles. They are used every time that you solve a length on the unit circle. And it may be of use in the future to see how these isosceles triangles relate to the unit circle and also how the height and width of the arc segments compare to the radius.

This should be explored further.

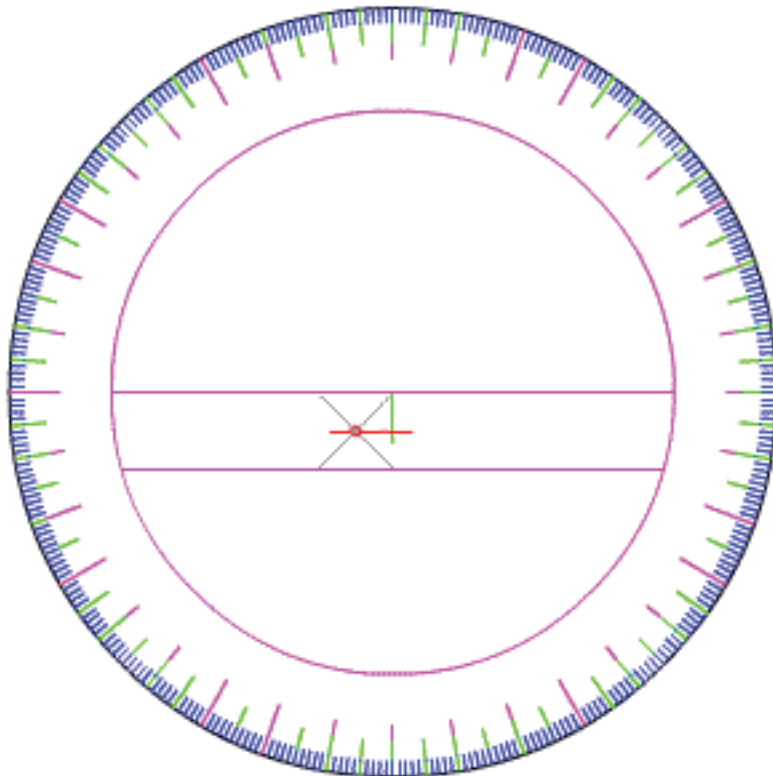
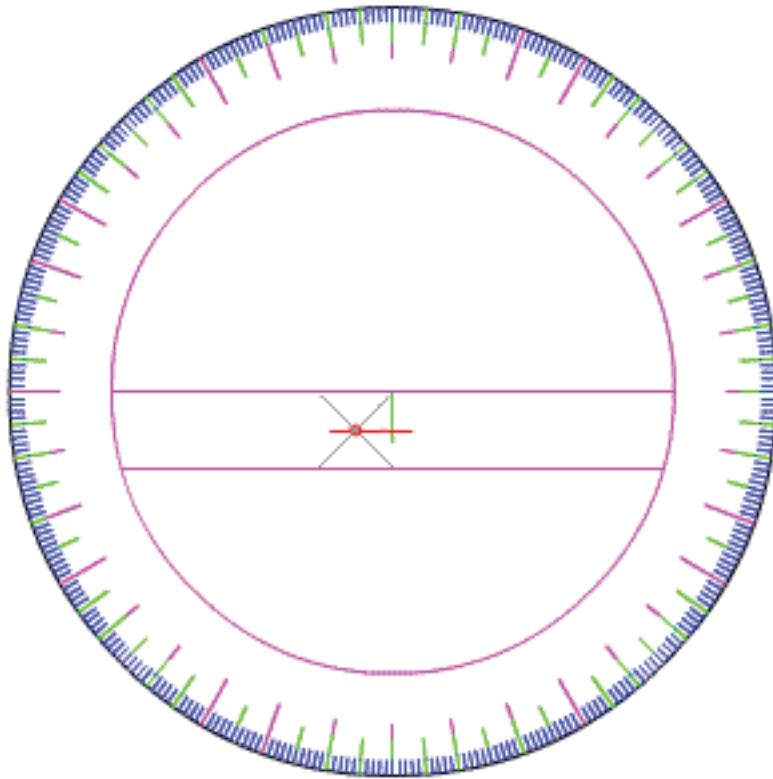
A circle is really made up of many isosceles triangles. They are used every time that you solve a length on the unit circle. And it may be of use in the future to see how these isosceles triangles relate to the unit circle and also how the height and width of the arc segments compare to the radius.

This should be explored further. And that is what the message board is for. We will solve some problems together. But first here is a simple drawing tool design I drew. It is very simple. It consists of three movable scales and a protractor. For now it can be printed on clear plastic and then pin the scales together. I want to design it with the pin removable so that the scales can be changed and also have a notch for a pencil to mark the radius.

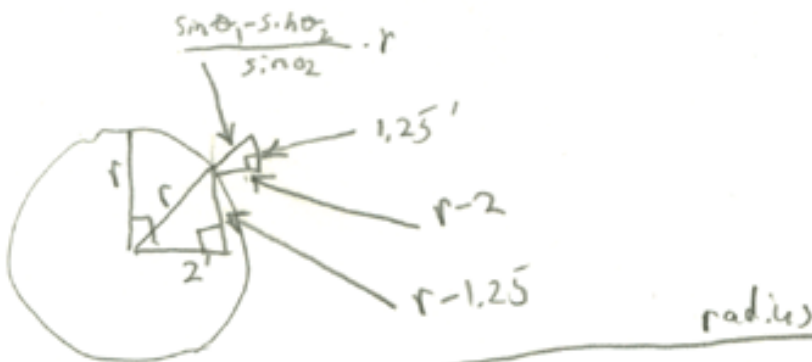
Here are the tools printable versions: If the file fails to download just right click on the mouse and select save target as:

Instructions are below:

1. Print on thick paper or on clear plastic and cut out. Be sure to cut out the center of the protractor. Basically just cut along the lines.
2. Line up the right edge of the scale (the side that is marked) with the green vertical line
3. Line up the red horizontal line of the scale to the red horizontal line of the protractor.
4. Pin in place and you are ready to construct circular arcs using the three radii (scales).



02-23-2004



$$(r-1.25)^2 + 2^2 = r^2$$

$$r^2 - 1.25r - 1.25r + 1.5625 + 4 = r^2$$

$$r^2 - 2.5r + 5.5625 = r^2$$

$$-2.5r = -5.5625$$

$$r = +2.22512$$

θ

$$r \cdot \theta = 2.5$$

$$2.22512 \cdot \theta = 2.5$$

$$\theta = \frac{2.5}{2.22512}$$

$$\theta = 1.12534910477$$

$$\approx 1.1253$$

check

$$\cos\left(\frac{\pi}{2} - \theta\right) \cdot r = 2'$$

$$\cos(0.4454 \text{ rads}) \cdot 2.22512 = 2'$$

yes it checks $2=2$

Reply for Snyder
(c) S. M. S. S. S.



$$A = \frac{1}{2} r^2 (\theta - \sin \theta)$$

Area

$$\begin{aligned} A &= \frac{1}{2} (2.22512^2) \cdot ((1.1253 \cdot 2) - \sin(1.1253 \cdot 2)) \\ &= 2.47557 - 1.472903 \\ &= 3.646288 \\ &\approx 3.65 \end{aligned}$$

Area of Door

$$\begin{aligned} &\text{Length} \times \text{width} + \text{area of arc} \\ &8 \times 4 + 3.65 \\ &32 + 3.65 \\ &= 35.65 \end{aligned}$$

SCos and SSin

The following math problem is a simple discovery I made when trying to find the radius when you only know the change in the sine or cosine. It comes from a problem I was working on in my trigonometry class at Point Park College. Since that time I have spent some of my free time solving the problem. I never saw this solution in any trig reference that I have come across. So to the extent of my knowledge it is original. I am still trying to find applications for this solution.

I have found that this new segment that is created forms a rectangle with two sides being the length of the radius (if taken from the $\cos = 1$ or $\sin = 1$). If the perpendicular is taken from $\cos 0$ and the second angle is 45 degrees then a square is formed with all four sides equal to the radius. I will explain more over the next pages.

But I hope you enjoy this math theorem I made up. Check it for any errors and contribute to any new discoveries or applications. I know the equation is complex. I tested it with sample values and it appears to work every time. If you have any questions just post a message on the message board or contact me through email.

We start with a unit circle.

It is known that as r (the radius) moves at the different angle the $[\cos(\theta) * r]$ or “the length of r in the x direction” changes.

The \cos decreases between 0 and 90. This information is widely known and documented.

But what would happen if we took a perpendicular (orthogonal) line from the $[\cos(\theta_1) * r]$ or “the value of “ x ” at θ_1 ” and wanted to find how much longer “ r ” would have to be at “ θ_2 ” to reach the same distance in the “ x direction?”

Theorem:

Given θ_1 and θ_2 are within 90 degrees of each other
and $\cos(\theta_1) \geq \cos(\theta_2)$

then:

The length of the segment (“ L ”) (which is at the same angle of θ_2) is given by the equation:

$$[(|\cos(\theta_1)| - |\cos(\theta_2)|) / \cos\theta_2] * r$$

We will call this length or distance “ L ” or “the length of the segment added to radius “ r ” to maintain the same horizontal “ x distance”.

So

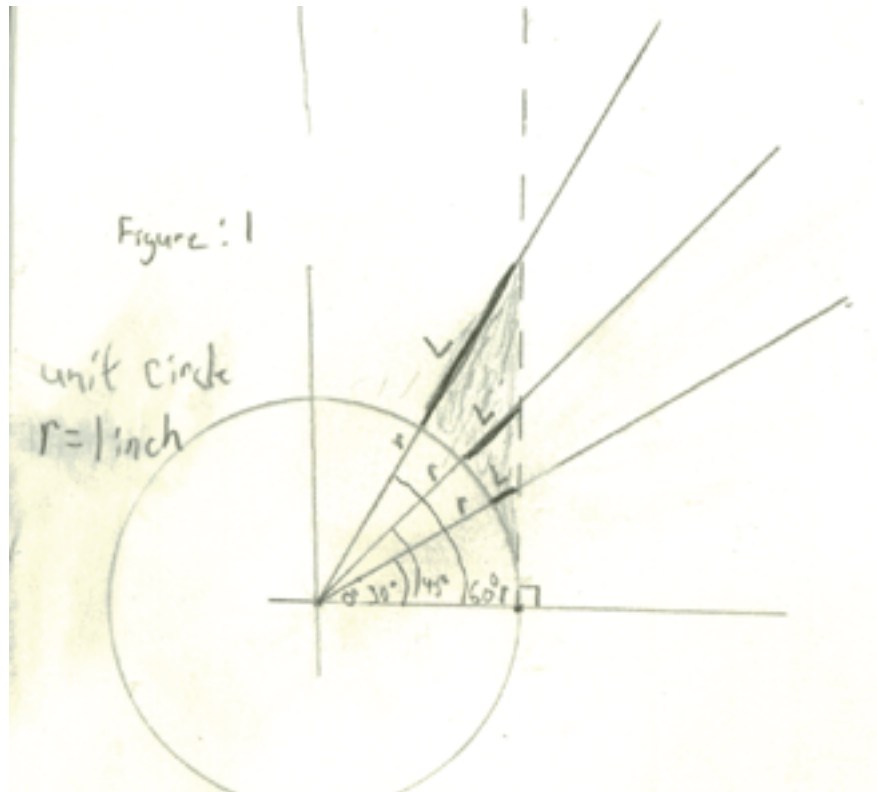
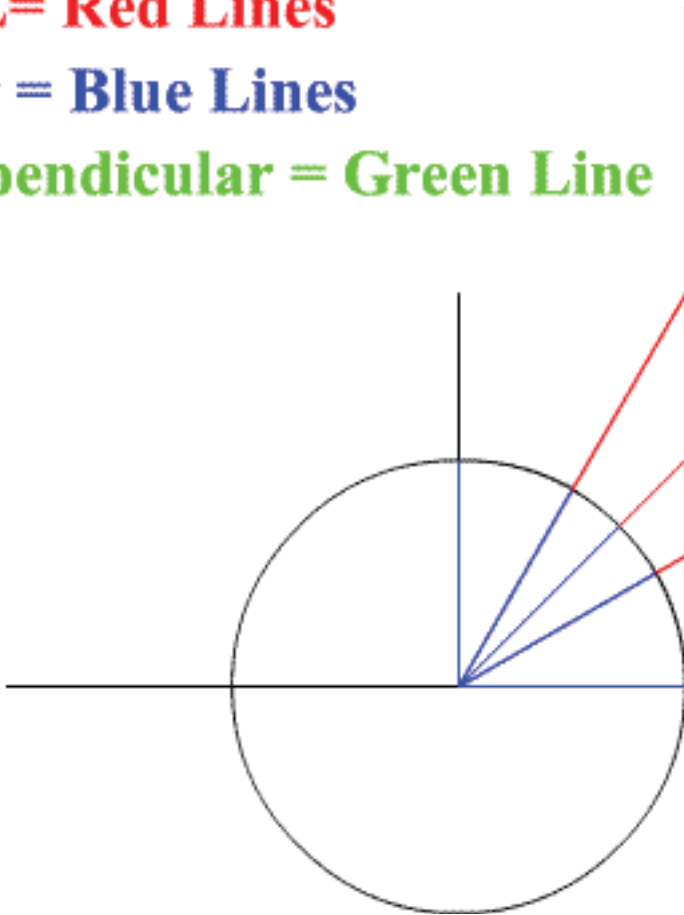
“the length of the original segment” or “ r ” + “the length of the newly found segment L ”

$$r + [(|\cos(\theta_1)| - |\cos(\theta_2)|) / \cos\theta_2] * r]$$

L= Red Lines

r = Blue Lines

Perpendicular = Green Line



This equation can be applied to the other quadrants. Just be sure that the $\cos(\theta_1) \geq \cos(\theta_2)$

We will start by comparing this new length "L" by the special angles of 30, 45, and 60 degrees.

On a unit circle with:

let $\theta_1=0$

let θ_2 =the current value in degrees

let $L = [(|\cos(\theta_1)| - |\cos(\theta_2)|) / \cos\theta_2] * r$

Value in deg	cos	$ \cos(\theta_1) - \cos(\theta_2) $	$(\cos(\theta_1) - \cos(\theta_2)) / \cos\theta_2$	L	L+R	$\cos(\theta_2)*r + \cos(\theta_2)*L$
0	1	0	0	0	1	0
30	.866	.134	.155	.155	1.155	1.134
45	.707	.293	.414	.414	1.414	1.293
60	.500	.500	1.000	1.000	2.000	1.500
90	0	no value	no value	no value	no value	no value

As θ_2 approaches 90 degrees the length of L and L+R approaches ∞ "infinity"

Similarly for the Sine:

We start with a unit circle.

It is known that as r (the radius) moves at the different angle the $[\sin(\theta) * r]$ or "the length of r in the y direction" changes.

But what would happen if we took a perpendicular (orthogonal) line from the $[\sin(\theta_1) * r]$ or "the value of " y " at θ_1 " and wanted to find how much longer " r " would have to be at " θ_2 " to reach the same distance in the " x direction?"

Theorem:

Given θ_1 and θ_2 are within 90 degrees of each other
and $\sin(\theta_1) \geq \sin(\theta_2)$

then:

The length of the segment (" L ") (which is at the same angle of θ_2) is given by the equation:

$$[(|\sin(\theta_1)| - |\sin(\theta_2)|) / \sin\theta_2] * r$$

We will call this length or distance " L " or "the length of the segment added to radius " r " to maintain the same vertical " y distance".

So

"the length of the original segment" or " r " + "the length of the newly found segment L "

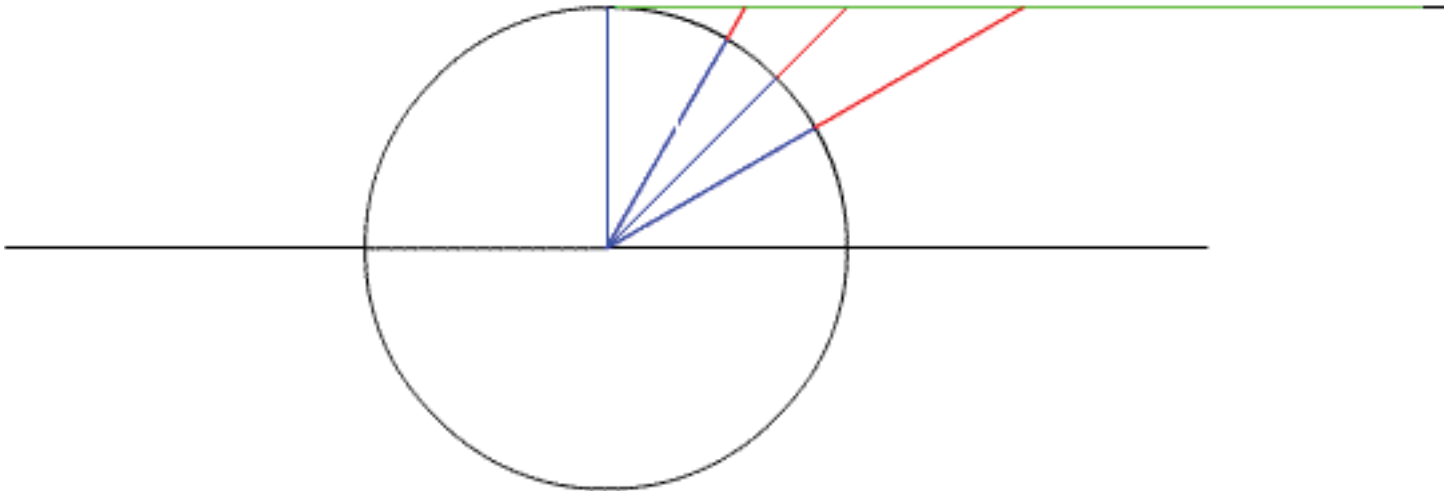
$$r + [(|\sin(\theta_1)| - |\sin(\theta_2)|) / \sin\theta_2] * r$$

This equation can be applied to the other quadrants. Just be sure that the $\sin(\theta_1) \geq \sin(\theta_2)$

L=Red Lines

r=Blue Lines

Green Line= Perpendicular



We will start by comparing this new length “L” by the special angles of 30, 45, and 60 degrees.

On a unit circle with:

let $\theta_1=90$ degrees

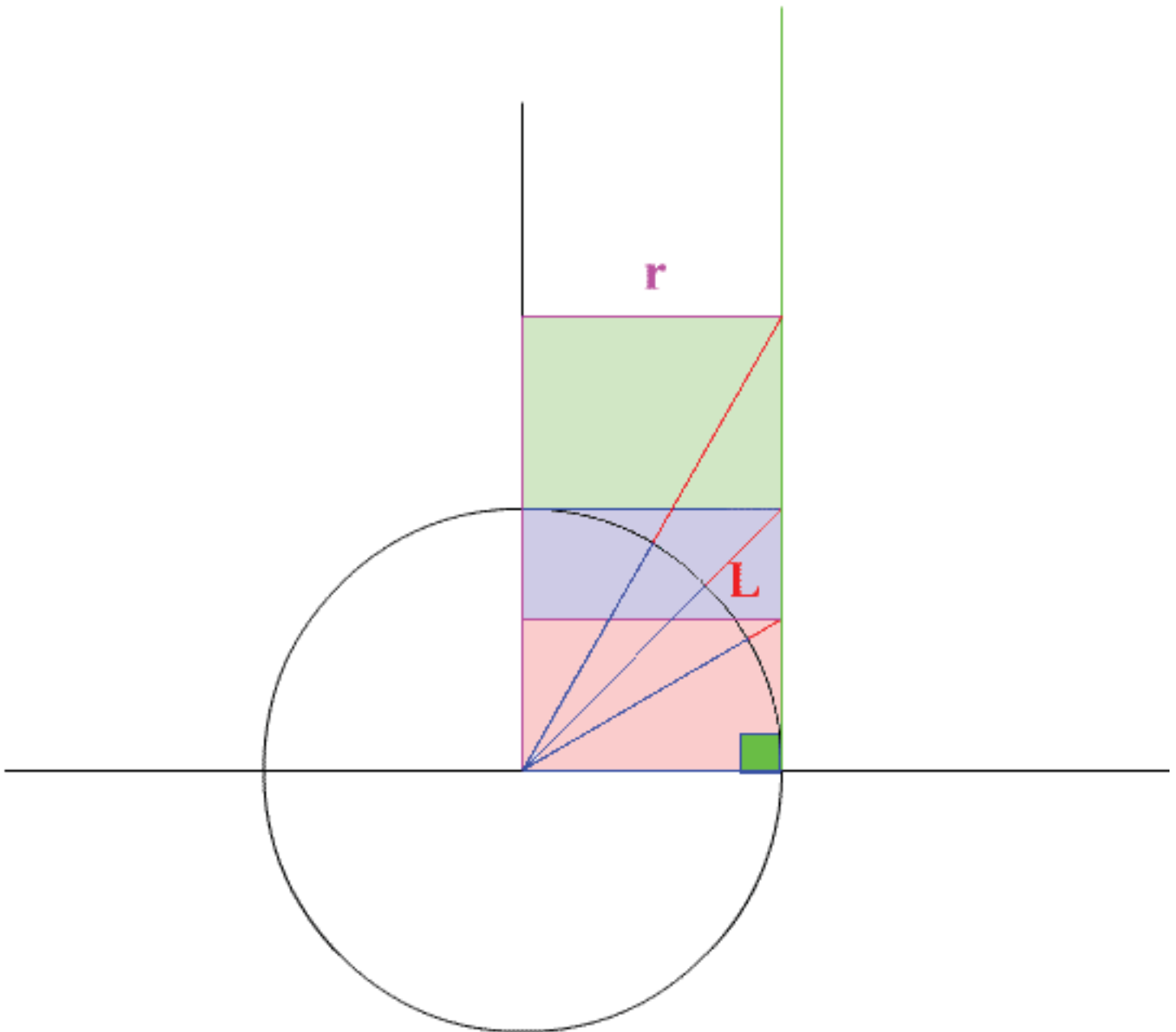
let θ_2 =the current value in degrees

let $L = [(|\sin(\theta_1)| - |\sin(\theta_2)|) / \sin\theta_2] * r$

Value in deg	sin	$ \sin(\theta_1) - \sin(\theta_2) $	$(\sin(\theta_1) - \sin(\theta_2)) / \sin\theta_2$	L	L+R	$\sin(\theta_2)*r + \sin(\theta_2)*L$
0	0	no value	no value	no value	no value	no value
30	.500	.500	1.000	1.000	2.000	1.500
45	.707	.293	.414	.414	1.414	1.293
60	.866	.134	.155	.155	1.155	1.134
90	0	1.000	0	0	0	0

As θ_2 approaches 0 degrees the length of L and L+R approaches ∞ “infinity”

These trigonometric theorems are more complicated than the usual cosine and sine functions, but there is a simple rectangle that is created. If the perpendicular is taken from the cosine, the rectangle has a distance in the "x direction" of the cos of θ_1 or a distance of r if the perpendicular was taken from 0 degrees or 180 degrees. The same can be said about the sine. If the perpendicular is taken from the sine, the rectangle has a distance in the "y direction" of the sin of θ_1 or a distance of r if the perpendicular was taken from 90 degrees or 270 degrees.



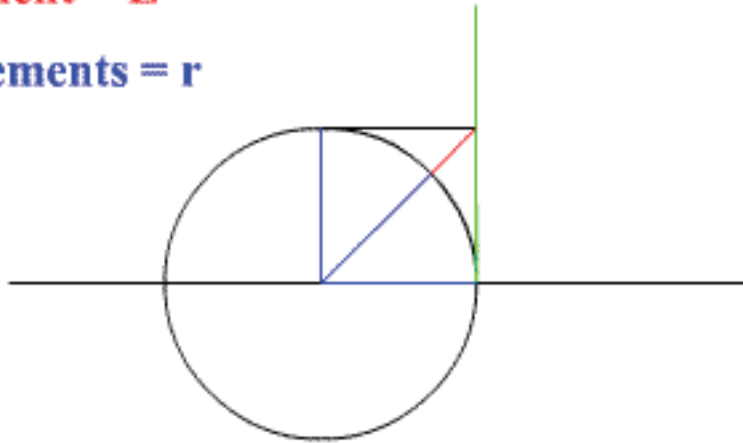
There is a special rectangle formed at 45 degrees with all four sides equal to the radius.

This is a square formed around the diagonal $L + r$

(where r is the radius and L is the length previously described in the previous equations)

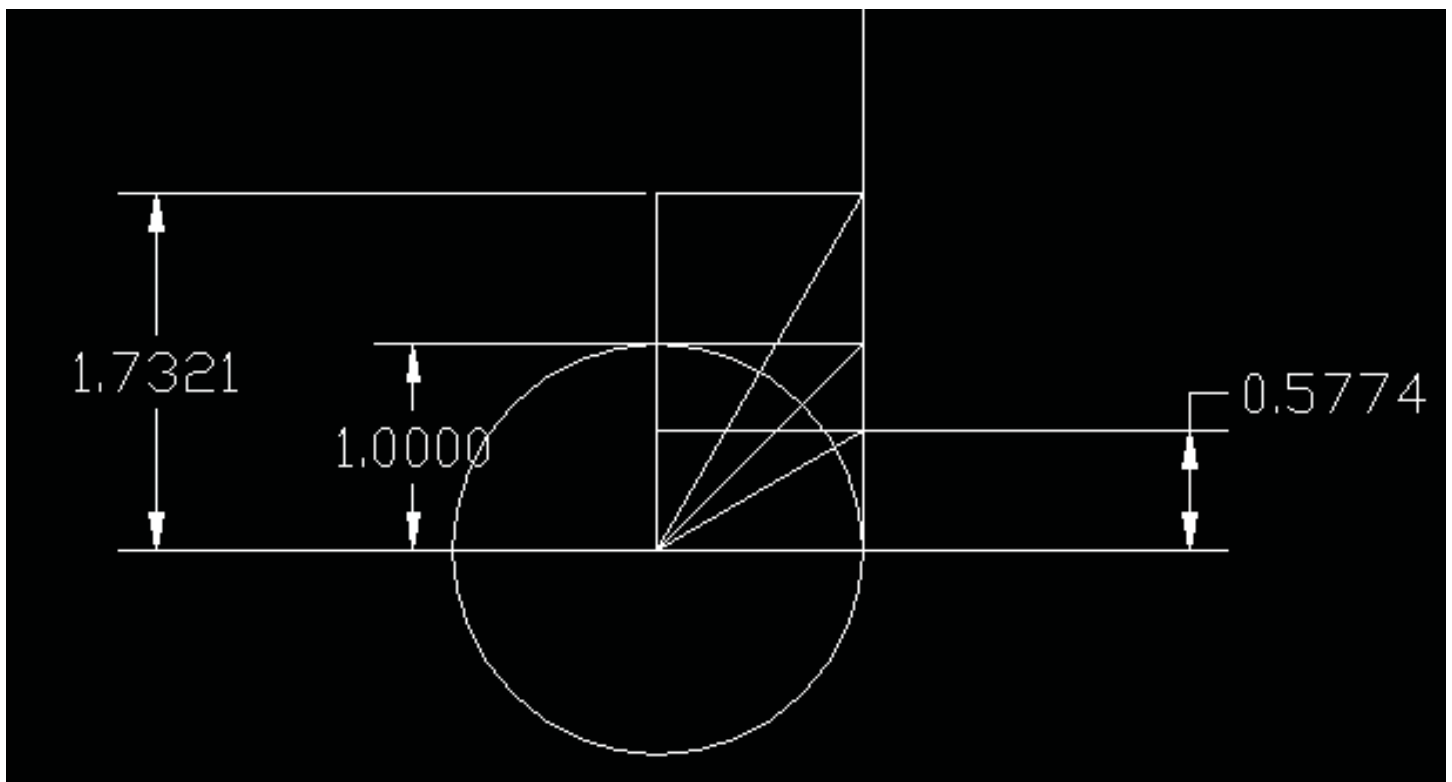
Red segment = L

Blue segments = r

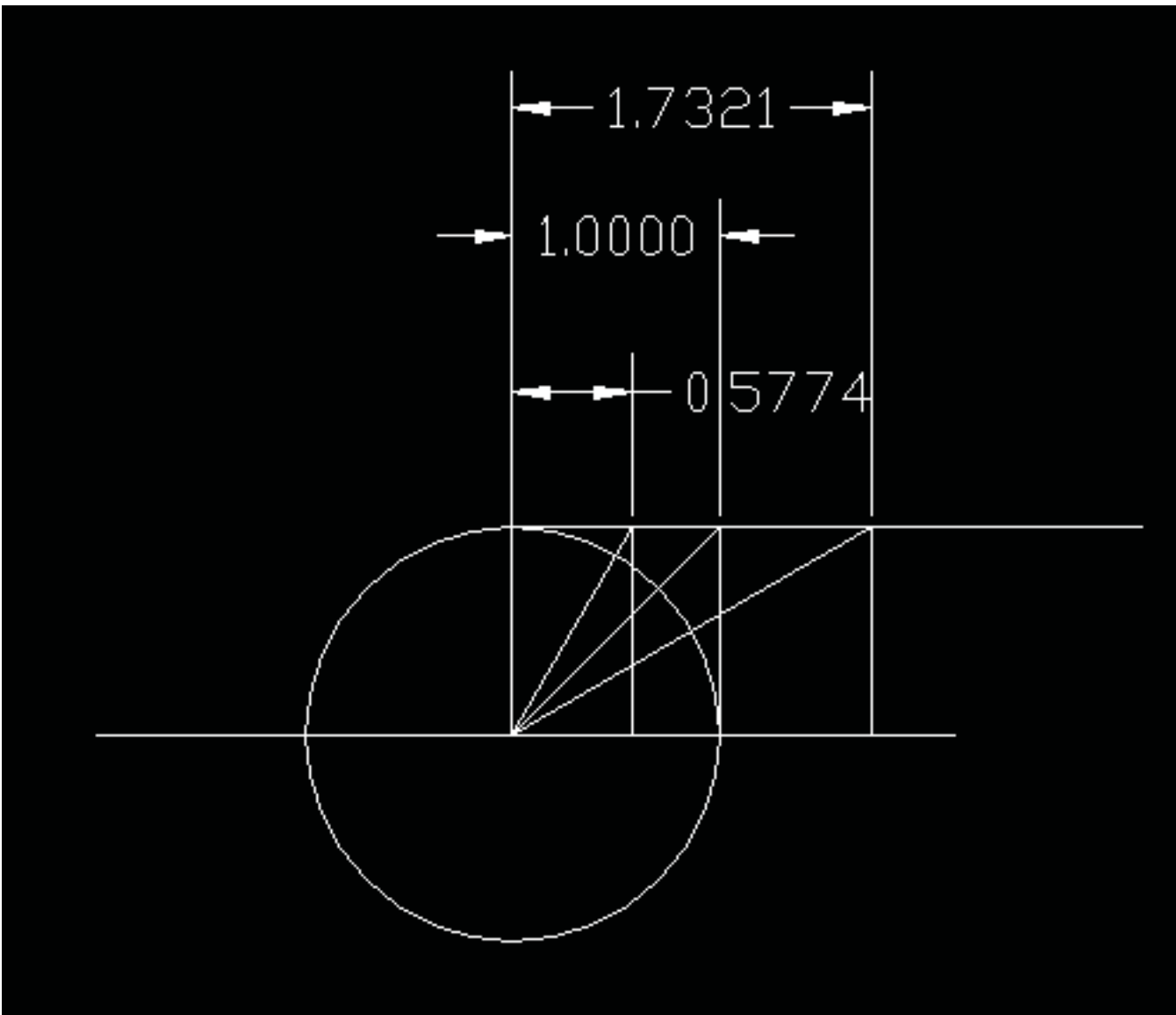


At 45 degrees a special rectangle is formed. Where all 4 sides equal the radius. This happens when the perpendicular is taken from both the sine and cosine!

The ratios (or proportions) of the height “y direction” for $L + r$ for the cosine. As follows:



The ratios (or proportions) of the height "x direction" for $L + r$ for the sine. As follows:



So what have we just discovered? That is a good question.

We have just found what length of segment would have to be added to the radius to maintain either the same cosine or sine. In other words, the length of the cosine or sine (depending on if you take where you take the perpendicular from) or "x distance" or "y distance" stays the same, but the length need to maintain that distance at an a greater angle increases.

First this is another tool in or arsenal of trigonometric theorems and laws. Secondly I believe if you add on to this technique you can solve angles by how much the cosine or sine changes. These theories are incomplete and a hard to find discovery possibly lies ahead.

That is why I have posted this math exercise. I want to get people to work on this idea and discover more about it. The message board is up and running and is awaiting intellectual interaction.

May the creative force be with you,
Trurl

Unknown Curves

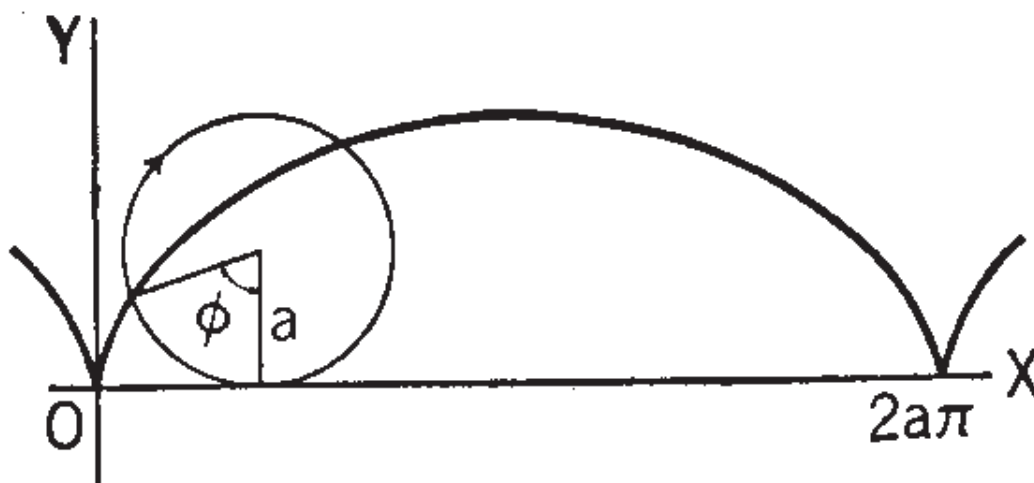
Suppose you are measuring an object that is mad of lots of curves. And you want a way to measure and evaluate the curves. But instead of heading to the computer first you are only armed with a set of drafting tools. You have a pair of triangles, a compass, a circle template, and a scale. In your pocket is a Swiss Army knife and some duct tape. And on a table in the room is some glue and some cardboard.

Your mission is to measure the sine, cosine, and tangent of any point that is on the object made of curves.

What we need is a starting point. What if we were to wrap a function across another function? In this case we have a circle and there are many circular functions. Perhaps if we were to roll the circle along the curves of the object, it would describe the curves. In other words, we will roll the circle like a wheel along the road of the curves of the object.

A circle is based on trigonometric functions. In fact one of the equations describing the circle is "a cos x + a sin x"

So the plan is to use a circle (which is moving like a wheel) to find the value of the sine and cosine of the curve. The first step is to find how a circle moves across a horizontal line. The curve that is formed is called the cycloid (ordinary case).



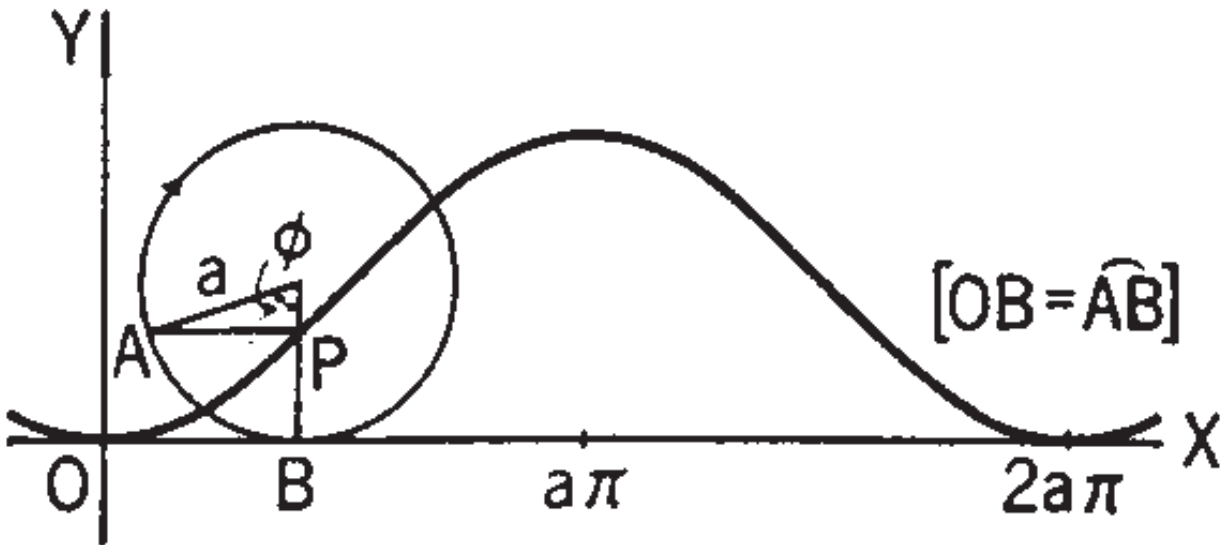
CYCLOID, ORDINARY CASE

$$x = a \text{ arcvers } \frac{y}{a} - \sqrt{2ay - y^2}$$

$$x = a(\varphi - \sin \varphi), \quad y = a(1 - \cos \varphi)$$

Well that gives us a curve that can be put into equation form, but the equation is difficult to work with. (it is difficult to work with but it may have use for something) But we are looking to draw the sin and cos out of a circle in motion. And then use those values to find the tangent. The trigonometric values will probably prove to be the most useful when describing the objects curves.

But there are variations to the cycloid. One being the companion to the cycloid. It is useful because the curve it creates is already like the sine and cosine. It is just out of phase.



COMPANION TO THE CYCLOID

$$x = a\varphi, y = a(1 - \cos \varphi)$$

The equation $x = a\phi$, $y = a(1 - \cos \phi)$ can be transformed very easily using a unit circle to say

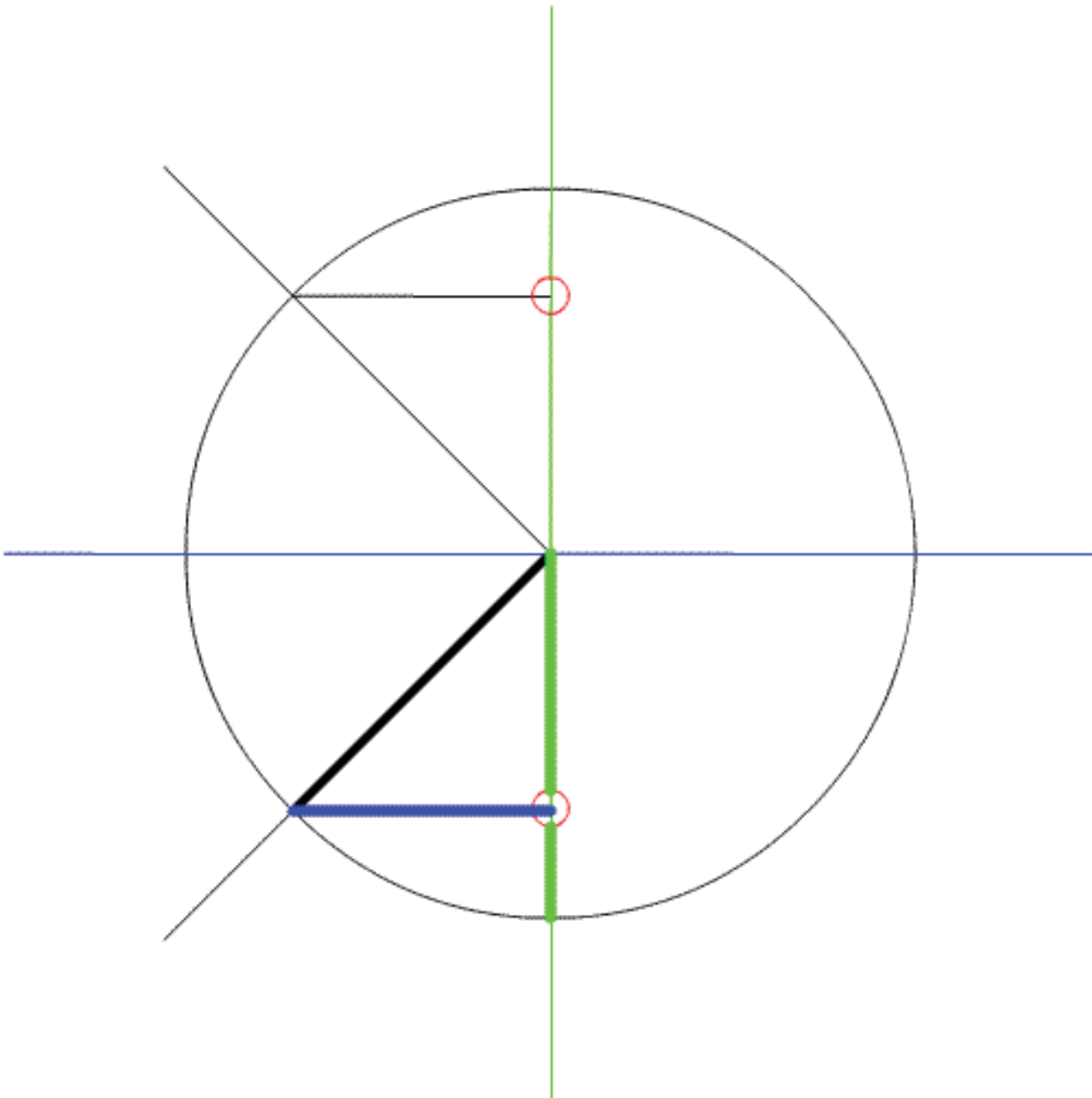
$$x = \phi, y = (1 - \cos (\phi + \pi/2))$$

the equation is now in the form of the sin.

So we measure an angle from the graph of the cycloid (point P) $\pi/2$ (90 degrees). We then draw a line that intersects with the circle. Then draw a line perpendicular to the line that touches the circle. The line that is perpendicular to the horizontal center line intersects the line just drawn is where the sine path lies. For a simpler picture refer to the drawing.

Note the position of this line is not stationary it changes by x distance $OB = \text{arc AB}$. So as the circle moves so does the position of the cycloids curve. If the point were stationary the curve formed would just be a variation of the ordinary cycloid.

The cosine is given by 1+ or - the vertical length to point "P". It is added if P is in the lower quadrant and subtracted if P is in the upper quadrant.

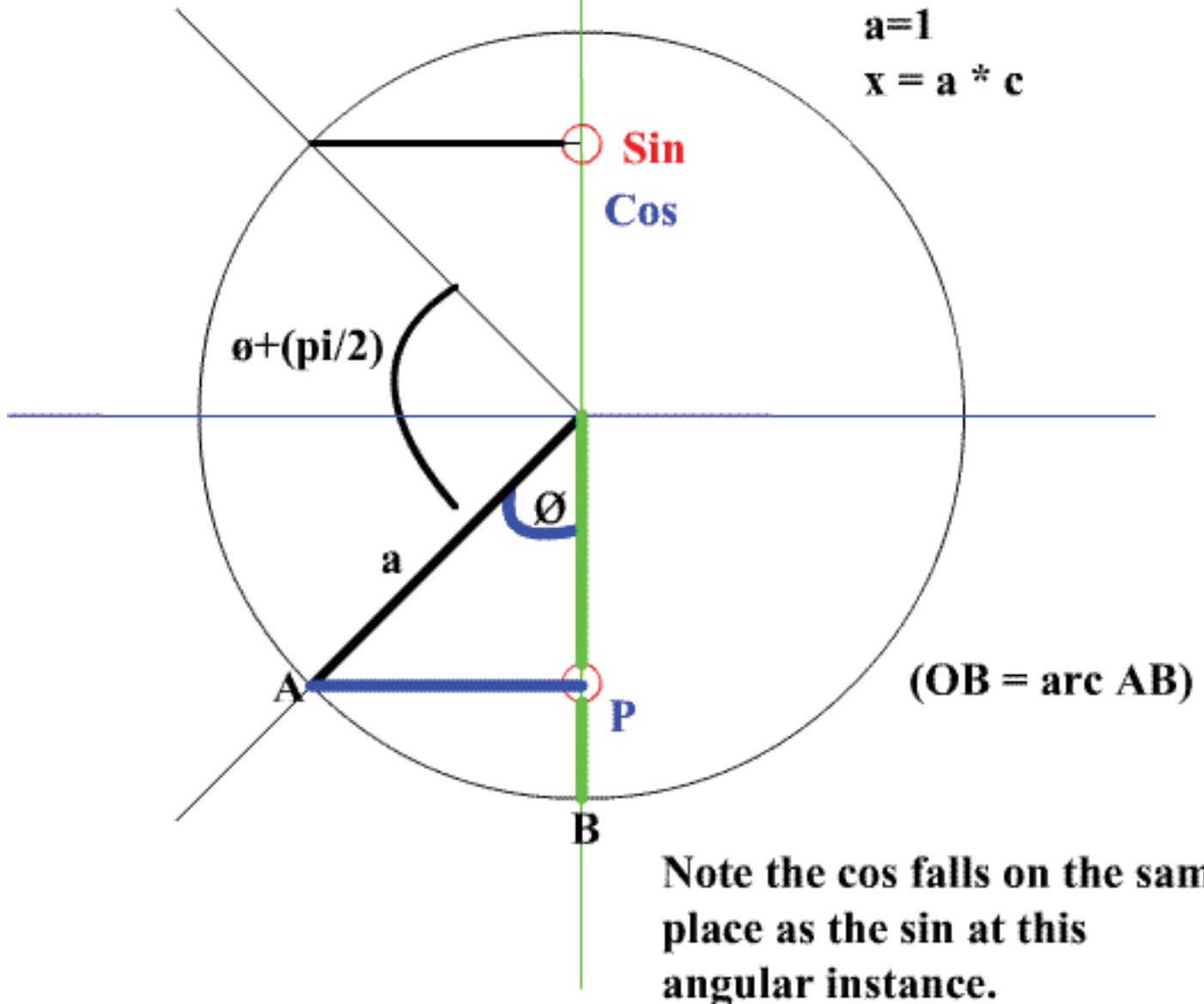


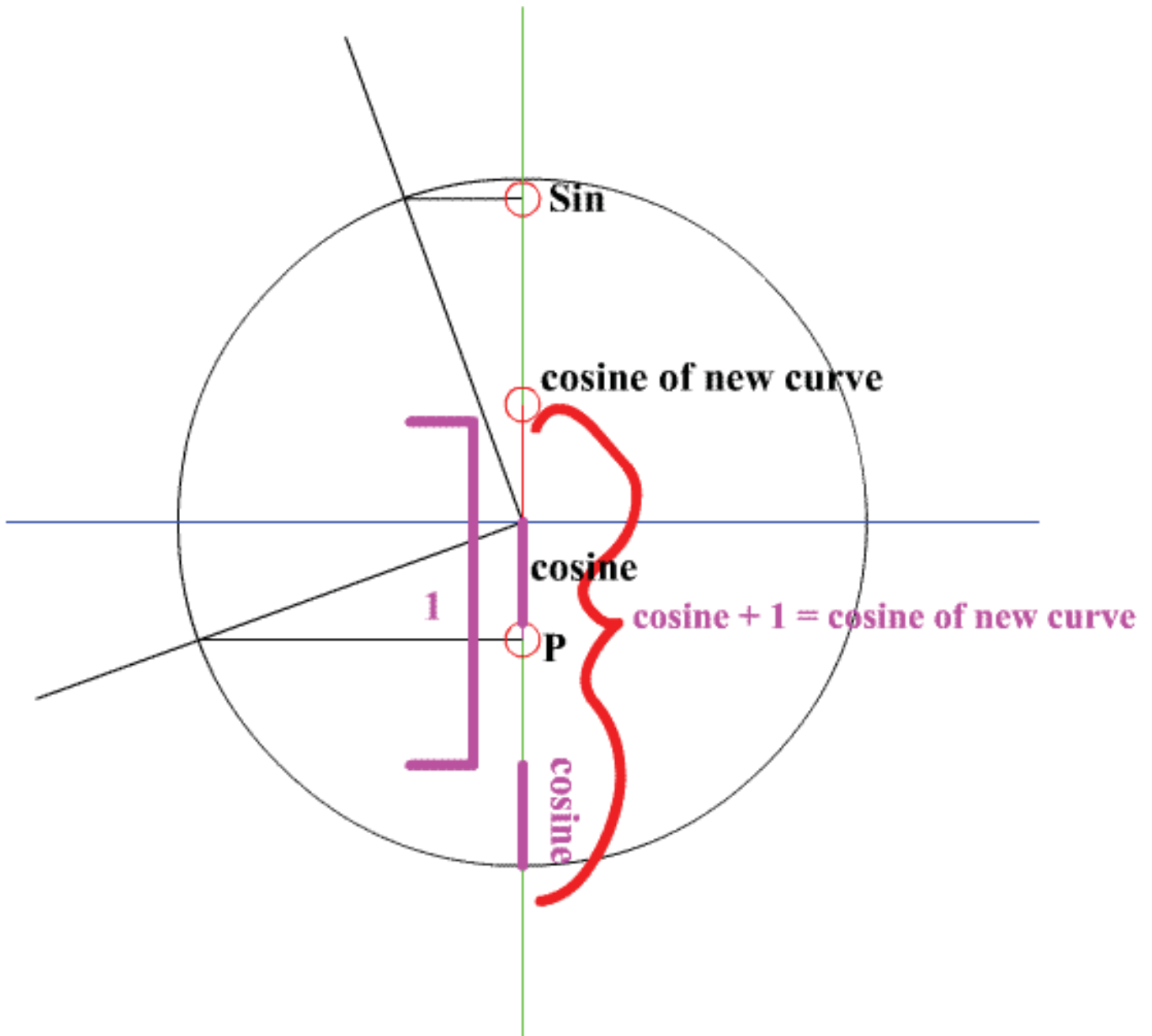
$$x = a \theta, y = a(1 - \cos \theta)$$

$$y(x) = \sin x = 1 - \cos(x + \pi/2)$$

$$a = 1$$

$$x = a * c$$





Now that we have two functions that are well known and easy to work with, we know follow the curve with the circle with a constructed instrument or mathematically. Notice that as the circle rotates that the sine and cosine stay within the vertical center line. (This should be explored more!) So it is possible to construct an instrument to make it easier to draw this curve.

Now that the sine curve is drawn across the curves of the unknown object, it is time to compare the curve formed by the circle to the original sine curve.

Take the y values of the curved object and subtract it from the value of the sine curve we drew. Then take this newly modified curve of the sine and compare it to the actual sine curve itself. (For ease of use you can just run your circular instrument we created across a horizontal line.)

Add one to the value of the true sine curve to eliminate negative numbers. Subtract the only positive number sine curve from the curve we wrapped around the unknown curve object. Restated, subtract the sine curve from the curve formed by the rotating circle.

Repeat the steps for the cosine. And once the sine and cosine is found you can use the formula $\sin/\cos = \tan$ to find the tangent.

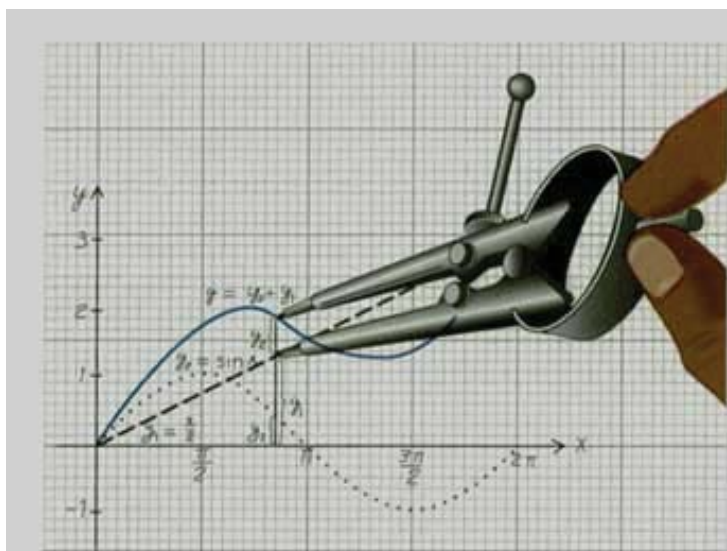


FIGURE 1
Addition of ordinates

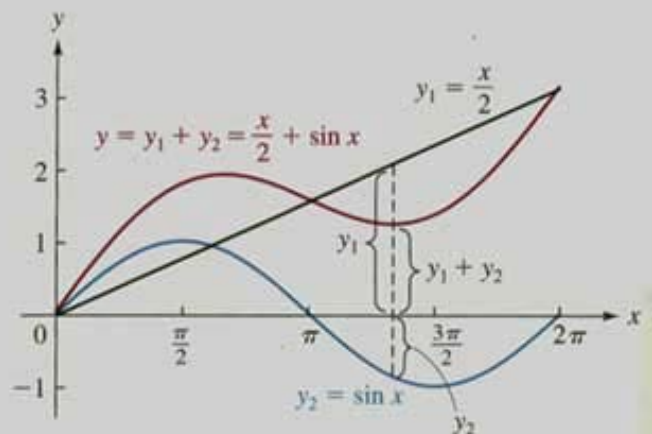


FIGURE 2
Addition of ordinates

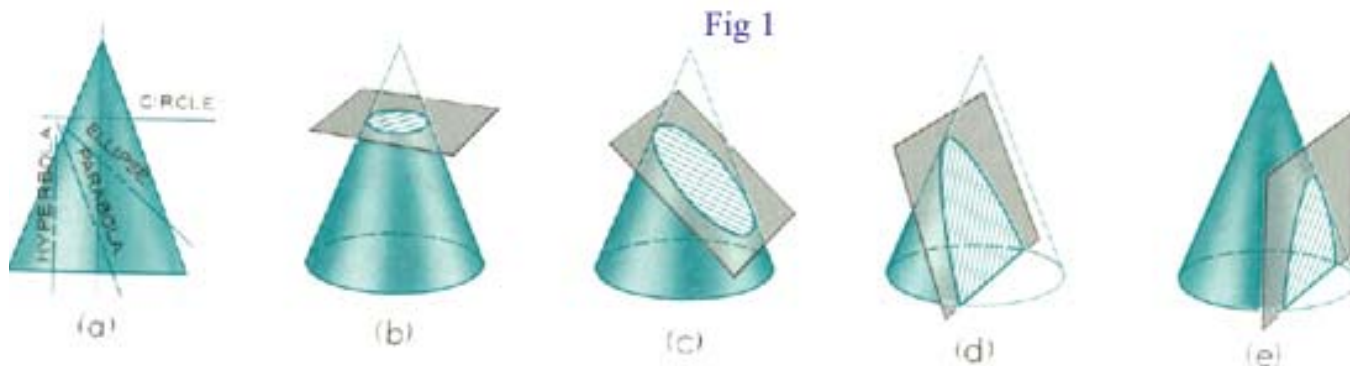
References: pic 1 and 2 from: "Rinehart Mathematical Tables, Formulas and Curves", Larsen, Rinehart 1955

pic 6 from: "Analytic Trigonometry with Applications Sixth Edition" Barnett, PWS Publishing Company 1995

Extraordinary Ellipse

We are already familiar with the “unit circle.” We have already used it to define circular functions such as the graph of the sine or cosine.

We also know from algebra that a circle is a conic section. Picture two cones with their points facing each other. A circle is formed by slicing the cone with a plane that is perpendicular to the cones. See figure 1.



Pic excerpt from: “Technical Drawing” 10th edition, Prentice Hall: Giesecke

But what if we could describe some of the other conic sections using the sin or cosine or convert the unit circle to match the properties of the new shape.

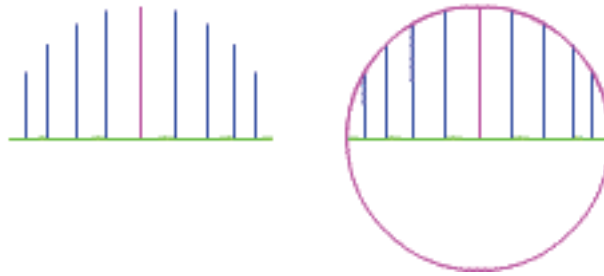
We will be describing the ellipse using the sine and cosine. It is another conic section that is formed by using a cutting plane at an angle to one of the cones discussed earlier. See figure 1.

The idea to describe an ellipse by the sine and cosine comes from my work with vectors. I believe that ellipses and circles can be used to find many things with problems dealing with velocities, rotation, and forces. In fact, ellipses and circles are already used all the time to explain and solve these problems. But perhaps there exists an approach to solve such problems that was hidden in plain sight.

Before the equation is given we are going to draw the ellipse.

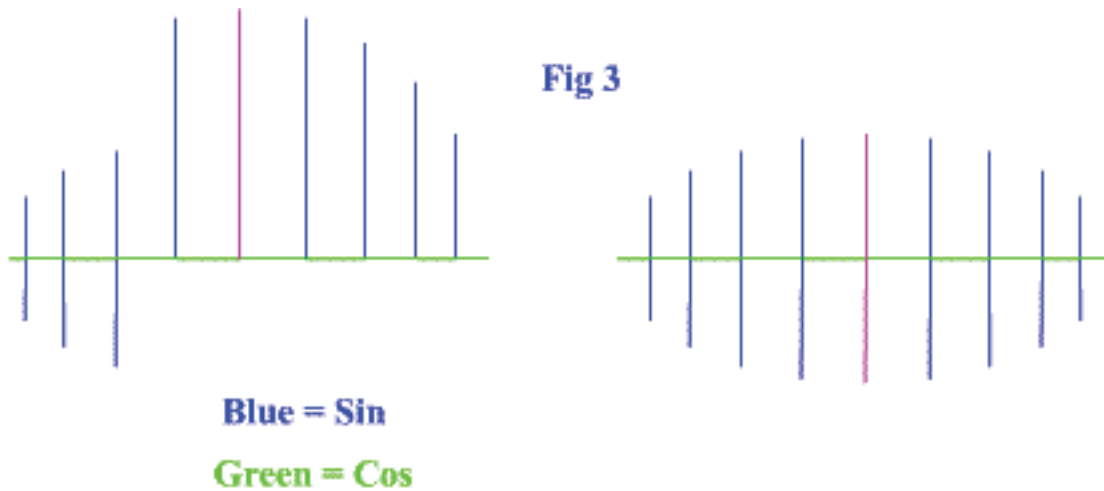
Start by drawing the beginning of a circle with the cosine being the distance in the x direction and the sine being the height in the y direction. Draw only in the first and second quadrant. The lines should be drawn upwards. See figure 2.

Fig 2



We just drew a circle for reference. Now draw a line 2 units along the coordinate plane and place its center at zero. From here draw the cosine and sine distance again but keep the lines centered vertically along the 2 unit long line. Restated, put the lines at a distance of the half the sine above the original 2 unit line and half the sine below the 2 unit line. Do this while maintaining the distance of the cosine in the x direction. See figure 3.

Fig 3



Now look at what you just drew. It has an x distance of the cosine and a y distance of half the sine. But it is also the structure behind an ellipse! And not just any ellipse. This ellipse describes the sine, cosine, and tangent. And with a little more investigating we see that it is the structure of the ellipse is a segment of the length of one. This is a similarity it shares with the unit circle. See figure 4 and figure 5.

Fig 4

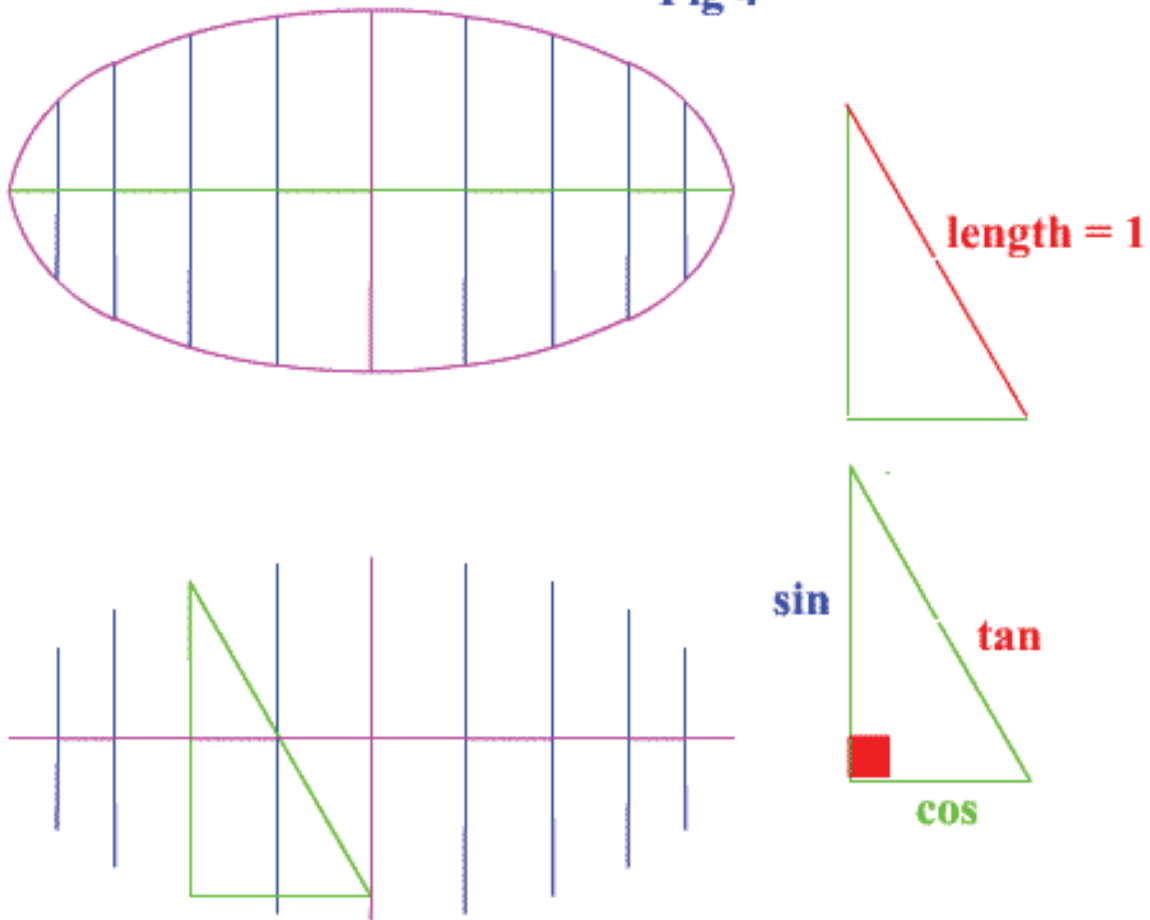
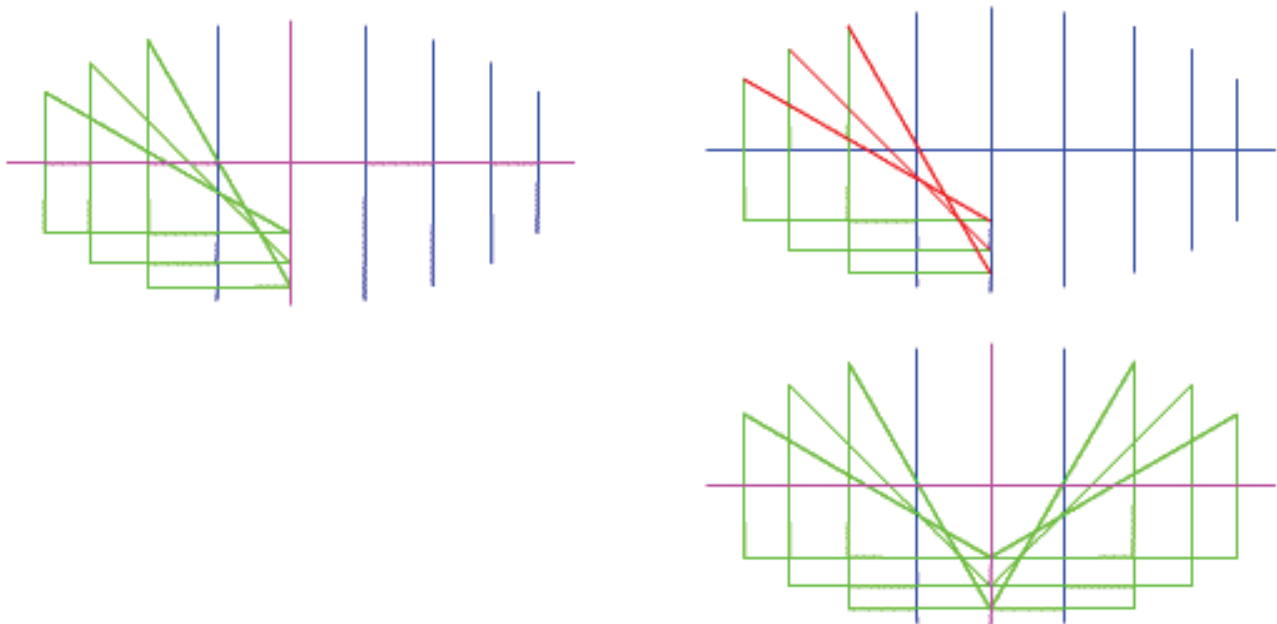


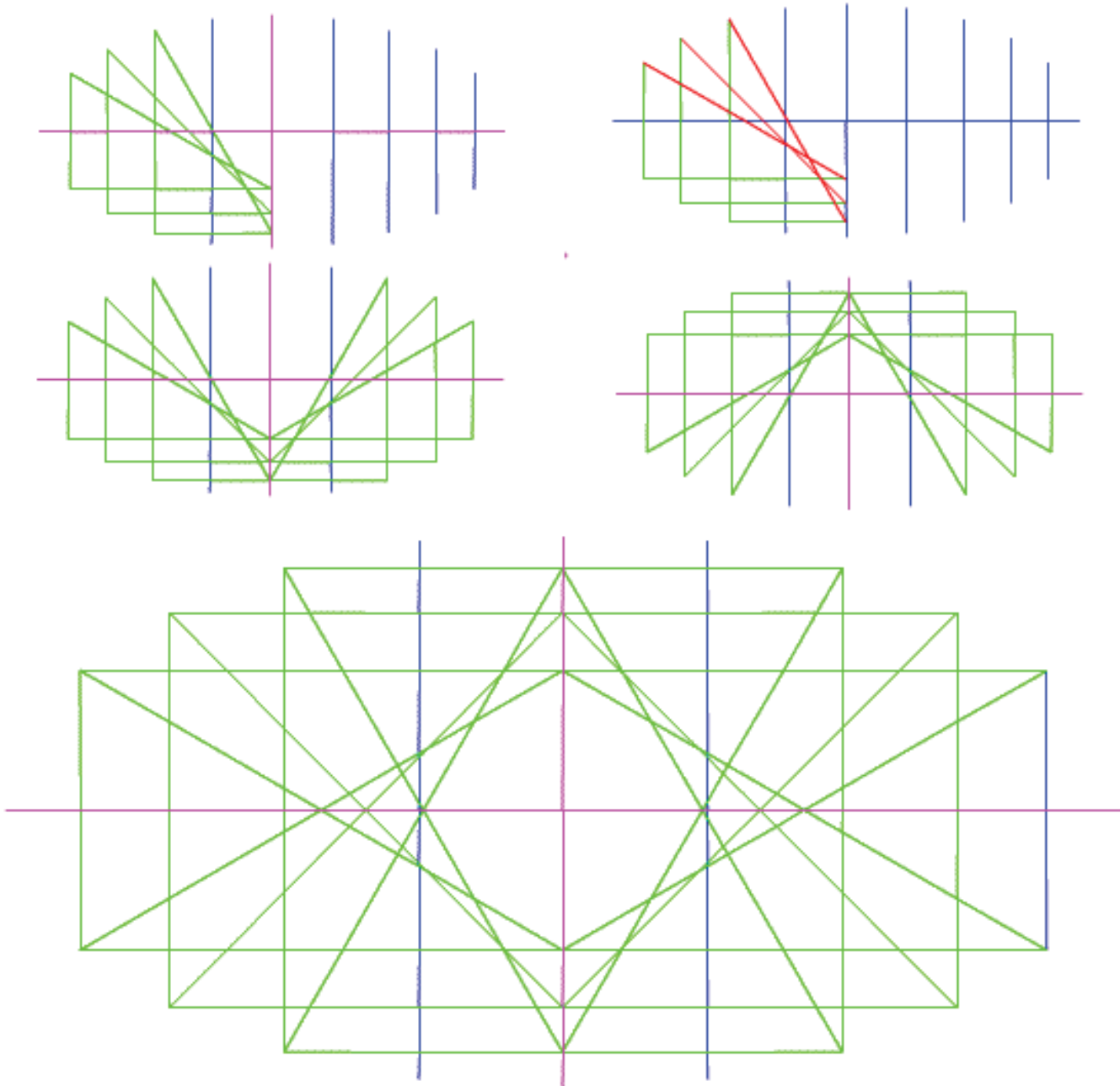
Fig 5



So we have just found that a bunch of triangles (triangles that are formed by the sine and cosine) can be placed to form and mathematically describe an ellipse. See figure 6 for the equation of an ellipse

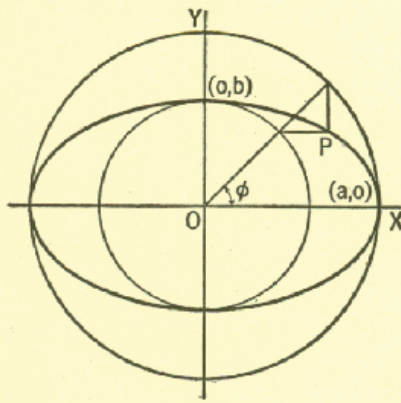
The triangles define the ellipse. The hypotenuse of those triangles has a value of 1. With the cosine in the x axis and the sine centered along the y axis the line will continue to rise up the minor axis until it reaches the top to the line (in $1 - \sin/2$ increments).

Fig 6



The triangles form the orbitals of the ellipse. As figure 6 shows, drawing the triangles from one end of the minor axis to the center of the minor axis is enough to complete the ellipse. It makes an awesome and complex shape.

Fig 7



ELLIPSE

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$x = a \cos \varphi, \quad y = b \sin \varphi$$

pic excerpt from: Rinehart Mathematical tables, formulas and curves, Larsen, 1955

So lets plug the numbers into the formula and see if it checks.

$$(x^2 / a^2) + (y^2 / b^2) = 1$$

$$((\cos x)^2 / a^2) + (((\sin x) / 2)^2 / b^2) = 1$$

You could try all the values of the cosine and sine. I solved this problem graphically and only using numbers to check the result. For now instead of a formal proof we will use numbers to test the findings.

$$((\cos x)^2 / a^2) + (((\sin x) / 2)^2 / b^2) = 1$$

$$(\cos x)^2 / 1^2 + ((\sin x) / 2)^2 / 0.5^2 = 1$$

$$\cos^2 x + (\sin^2 x / 4) / 0.25 = 1$$

$\cos^2 x + \sin^2 x = 1$ which is also known as one of the Trigonometric Pythagorean Identities. This is one of the fundamental identities in trigonometry.

(Note remember this measurement is from the center of the major and minor axis of the ellipse to the corresponding orbital. And it has an angle of x value.)

So lets test it for our example:

$$(\cos^2 60) + (\sin^2 60) = 1$$

$$0.5^2 + 0.8660^2 = 1$$

$$0.25 + 0.75 = 1$$

Figure 8 explains the length that was solved for in the previous equations. The length is in blue and $\frac{1}{2} \sin$ is in the bold magenta. This equation is similar if the sine is graphed in the x axis and the cosine is the y axis. Everything is done in almost the same way, but instead of the slope of the triangles formed being the tangent, it is the cotangent.

Fig 8

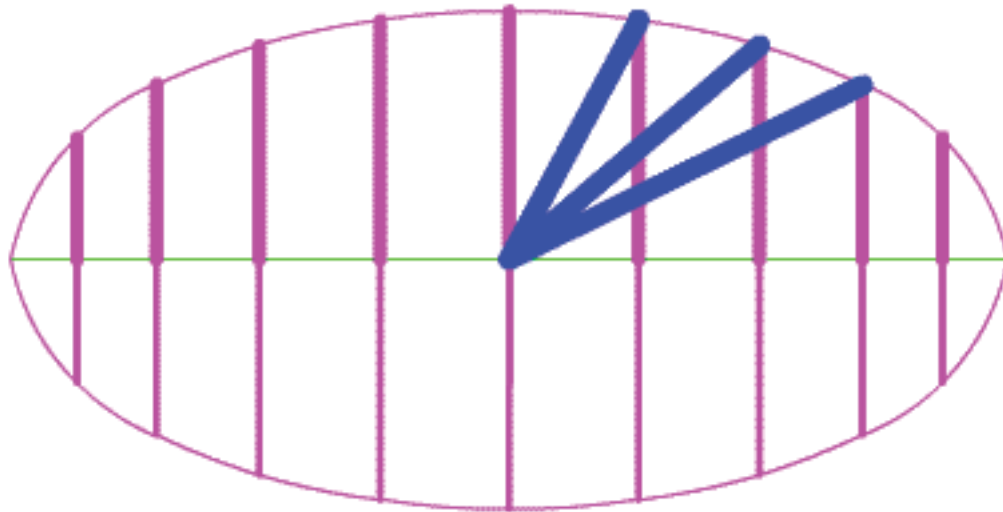
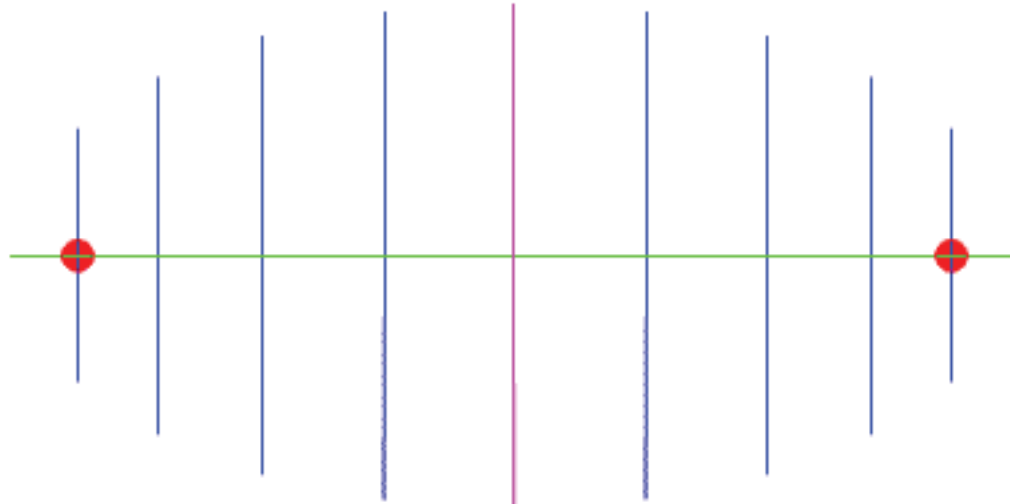


Figure 9 shows where the focus occurs. In this example (cos vs sin) it occurs at $\cos(30) * 1$ and $\cos(30) * -1$.

Fig 9



red dots = focus

So now we have an ellipse that has similar properties to that of a unit circle. (In having a way of describing the sine, cosine, and tangent.) But as we have seen this is an extremely extraordinary ellipse. It explains the sine, cosine, and tangent. But the wonder is in where the equation and shape fit the requirements to form an ellipse.

It is important to note that this equation and shape does not explain every single ellipse in the way that the unit circle explains all circles. This is a special ellipse and it might hold many uses to explain things the way the sine and cosine do, but ellipses consist of many different shapes and sizes unlike circles.

But the ellipse does have possible other applications and is unique enough to be worth studying.

For now have fun finding applications and problems where this ellipse belongs. And as always feel free to post messages on the message board.

Cell Tower

Intro: I am searching for new ways to use basic trigonometry and geometry to solve land navigational problems. (Perhaps even use the knowledge learned by the arc doorway problem.) Everything here is just based on the math I have learned. It is probably common knowledge with nothing newly discovered. Still it is interesting to think about. And the things we already know lead to the things that need uncovered.

In land navigation the path finder is equipped with a compass and a map. By shooting angles and knowing the distance found by pace counter or the maps scale, he finds his way.

To find position he simply finds two landmarks on his map. He shoots an azimuth or angle between one of the two points and his position. He then does the same with the second. Now he can use this angle and draw a line on the map from the landmark points. Where these two points cross is his position.

The situation is different when cell phone towers have to locate the user's phone. Here, since no angle is given, to find the user of the phone it would take 3 towers or points. The distance from the user to each tower is measured and where they intersect is the location of the user. Take a circle with a diameter of the distance and draw it around the position of the tower. With 3 circles the position is discovered, depending on the placement of the towers.

But what if there was a way to find the location using distance with only 2 towers? In fact, there is an easy way to do this with only knowing the distance and direction between the two towers.

Consider that the speed of the cell phones connect was taken by two towers. If you have two distances or circles around each tower, where they meet is the possible places for the cell phone user to be.

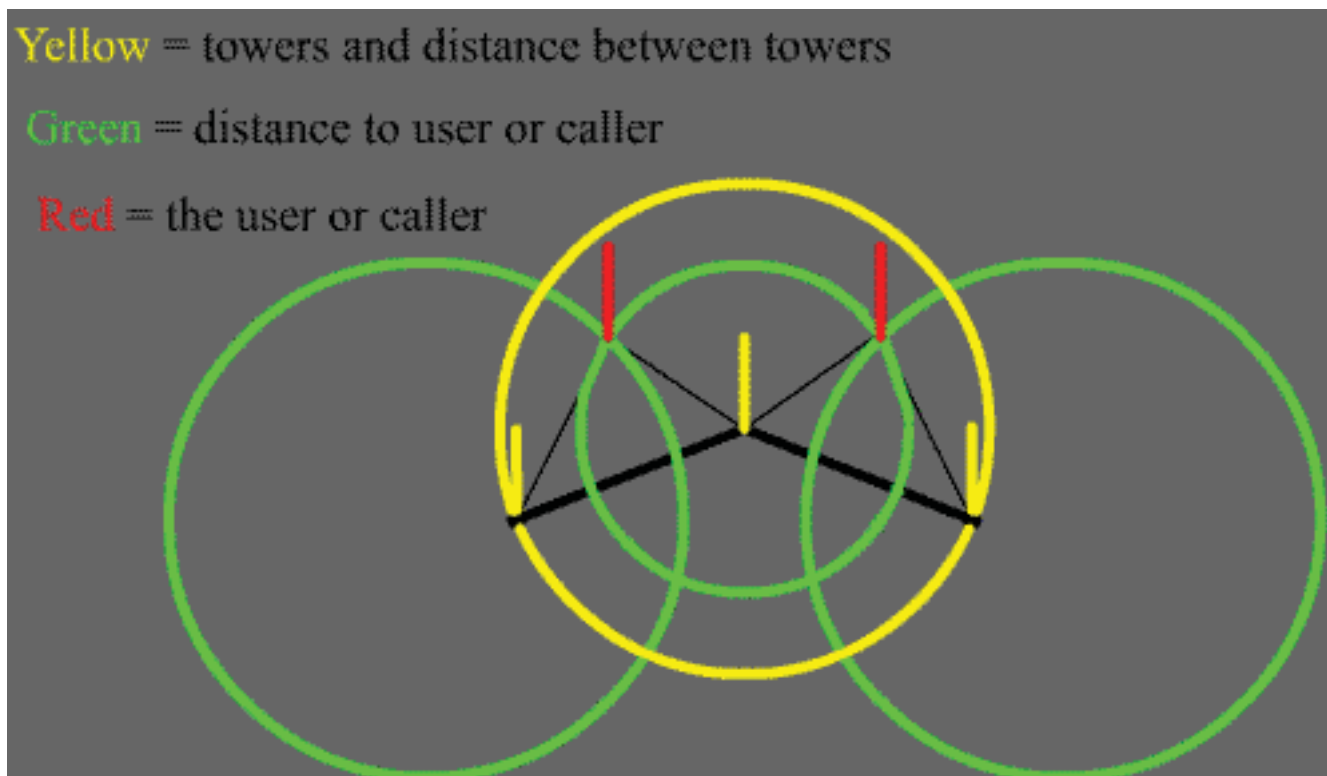
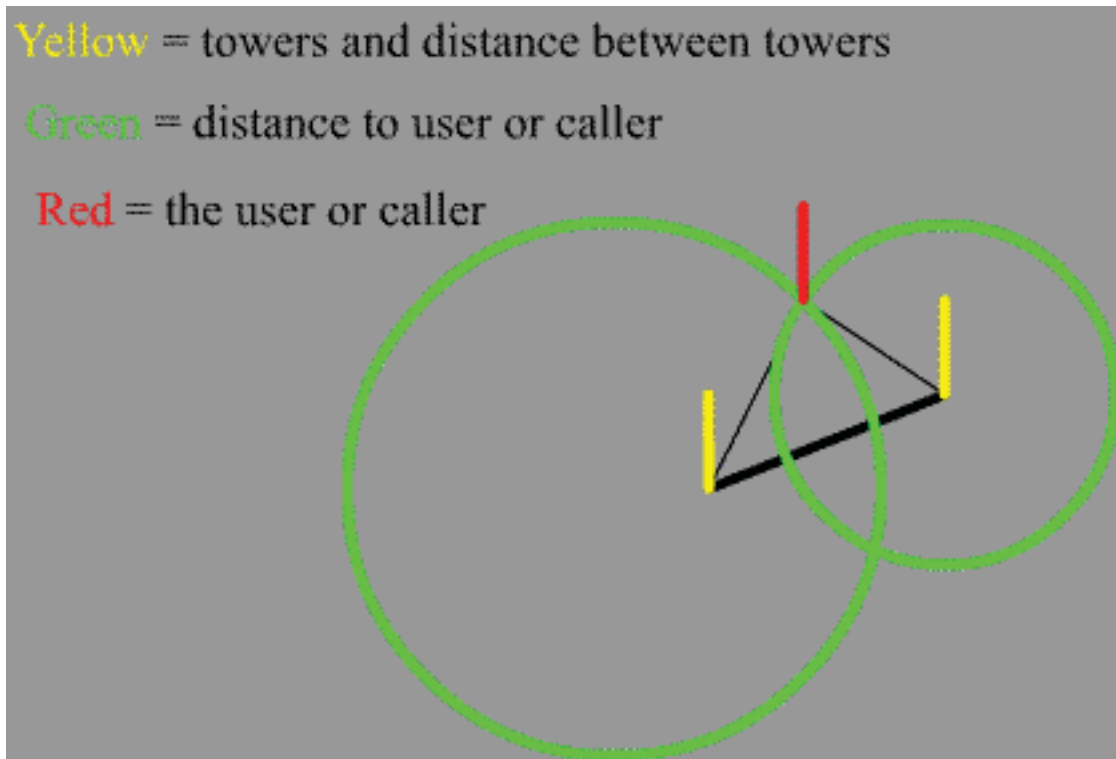
The circles meet in two places, and it is possible that the location could be mirrored creating four possible locations. But since the distance between the two towers is given it eliminates two choices leaving two choices remaining. Finally the correct choice is found by knowing the direction between the two remaining points or towers.

This leaves a triangle whose length of the sides are given and position is known. Now, the angles of the triangle can be solved by the law of cosines.

Conclusion: This problem doesn't really explain more than is already known. But it is an attempt to find little math cheats that can be used during land navigation. If you could discover something new that hasn't been discovered yet, then you would find a method that pertains not only to land navigation but movement in general such as programming a video game characters movement on the computer. I know that the methods in the field of navigation have been exhausted, but the intention is not to reinvent the wheel. The intention is to steer the wheel in a different direction. In other words, the attempt is to apply all the math knowledge that we have learned.

Figure 1 shows one possibility of the cell users position.

Figure 2 shows the other possible mirrored position. Once direction between the towers is given it eliminates the second possibility.



Mixture Problems

In calculus there are differential equations that have many practical applications in math and science. One problem that is often encountered is the "mixture problem." In these problems, a solid (usually salt) is dissolved in a liquid (water). The problem is to determine how much salt is in the tank at a given time.

Problem:

A tank contains 1000 liters of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?

Solution:

[salt already in tank at time t] - [summation of salt leaving the tank]

$$[0.1\text{kg} * 10 * t] - \left[\int_0^t 1\text{kg}/1000 * t * 10 \right]$$

$$= 1t - 1/200 * t^2$$

$$= 6 - 0.18$$

$$= 5.82 \text{ kg of salt after 6 minutes}$$

Warning!!! solution does not work

at least not yet ;)

Solution 2 The correct and proven method. (Instructor's Method)

Book Method

$$0.1 \text{ Kg/L} \cdot 10 \text{ L/min} = 1 \text{ Kg/min}$$

$$1000 \text{ Kg/L} \cdot 10 \text{ L/min} = \frac{s}{100} \text{ Kg/min}$$

$$S(t) = \frac{ds}{dt} = 1 - \frac{s}{100}$$

$$\int \frac{ds}{1 - \frac{s}{100}} = \int dt$$

$$-100 \ln \left| 1 - \frac{s}{100} \right| = t + C$$

Solve for S

$$\ln \left| 1 - \frac{s}{100} \right| = \frac{t}{100} + C$$

$$\left| 1 - \frac{s}{100} \right| = e^{t/100 + C}$$

$$1 - \frac{s}{100} = C e^{t/100}$$

$$s = 100 + C e^{-t/100}$$

Initial Value

Find C

$$s(0) = 0$$

$$C = -100$$

$$s = 100 - 100 e^{-t/100}$$

Problem and Solution #2 excerpt from:

Math 0230A Exam 2

Summer 12-week session 1999

University of Pittsburgh

Instructor Mr. Kissel

So now we have two equations that both give the same result. The first is formed by algebra using calculus only to sum the salt leaving the tank. And the second one formed by constructing a differential equation. This allows us to tinker with the equations and find new combinations. It also gives us a clue as to what the differential equation is calculating. It shows use the turning gears that would otherwise be hidden by the differential equation. If you can find a simpler equation to do the same task you have just simplified the steps.

$$\int^t (ds / (1-(s/100))) = \int dt = s = 100 - 100 * e^{-t/100} = [0.1\text{kg} * 10 * t] - [\int t \text{ kg}/1000 * t * 10]$$

Warning!!! does not work because solution 1 does not hold true

Solution #1 is inaccurate. The solution does not hold to be true. Even though it works for a few values (values that are small), it does not give an accurate picture of what is happening with the salt in the tank. But it makes the mathematician inside everyone wonder, what is the simplest way to solve this problem. The salt in the tank is summed for 0 to x, but it is the change of rate of the salt that determines x. Not all of the salt is available to be subtracted at a certain time. And not all the salt leaves the tank at a certain time. For example, the salt entering the tank in this problem is divided by 1000 and 10 of those 1000 leave the tank. So there is still a concentration of x-x/100. Then in x+1 there is a concentration of (x+1-(x/100))/100.

```
for n=0:1000000
x=(x+1)-((x+1)-(x/100))/100;
n=n+1;
end
x

x = 100.0000
```

```
for n = 0:99  
x=(x+1)-(x+1)/100  
n=n+1;  
end
```

```
for n=0:1000000  
x=(x+1)-((x+1)/100);  
n=n+1;  
end  
x = 99
```

In this problem x reaches a stable amount at 100. In other words high values of x reach a maximum value of 100kg/l.

This is hard to describe in an equation. It requires the natural logarithm. It is solved by a differential equation. My proposal is to find an equation that only by a single integral. As you can see the summation by the computer is simple, but it is difficult if possible to put in the form of a single integral. Or maybe there isn't just a single integral, but a different way of solving the problem.

```
for n = 0:99
x=(x+1)-(x+1)/100
n=n+1;
end
```

1. x = 0.9900	11. x = 10.3615	21. x = 18.8369	31. x = 26.5020	41. x = 33.4341	51. x = 39.7034	61. x = 45.3732	71. x = 50.5009	81. x = 55.1382	91. x = 59.3322
2. x = 1.9701	12. x = 11.2479	22. x = 19.6386	32. x = 27.2269	42. x = 34.0897	52. x = 40.2963	62. x = 45.9094	72. x = 50.9859	82. x = 55.5769	92. x = 59.7289
3. x = 2.9404	13. x = 12.1254	23. x = 20.4322	33. x = 27.9447	43. x = 34.7388	53. x = 40.8834	63. x = 46.4404	73. x = 51.4660	83. x = 56.0111	93. x = 60.1216
4. x = 3.9010	14. x = 12.9942	24. x = 21.2179	34. x = 28.6552	44. x = 35.3815	54. x = 41.4645	64. x = 46.9659	74. x = 51.9413	84. x = 56.4410	94. x = 60.5104
5. x = 4.8520	15. x = 13.8542	25. x = 21.9957	35. x = 29.3587	45. x = 36.0176	55. x = 42.0399	65. x = 47.4863	75. x = 52.4119	85. x = 56.8666	95. x = 60.8953
6. x = 5.7935	16. x = 14.7057	26. x = 22.7657	36. x = 30.0551	46. x = 36.6475	56. x = 42.6095	66. x = 48.0014	76. x = 52.8778	86. x = 57.2879	96. x = 61.2763
7. x = 6.7255	17. x = 15.5486	27. x = 23.5281	37. x = 30.7445	47. x = 37.2710	57. x = 43.1734	67. x = 48.5114	77. x = 53.3390	87. x = 57.7050	97. x = 61.6536
8. x = 7.6483	18. x = 16.3831	28. x = 24.2828	38. x = 31.4271	48. x = 37.8883	58. x = 43.7317	68. x = 49.0163	78. x = 53.7956	88. x = 58.1180	98. x = 62.0270
9. x = 8.5618	19. x = 17.2093	29. x = 25.0300	39. x = 32.1028	49. x = 38.4994	59. x = 44.2843	69. x = 49.5161	79. x = 54.2477	89. x = 58.5268	99. x = 62.3968
10. x = 9.4662	20. x = 18.0272	30. x = 25.7697	40. x = 32.7718	50. x = 39.1044	60. x = 44.8315	70. x = 50.0110	80. x = 54.6952	90. x = 58.9315	100. x = 62.7628

There is a series here.---- $x - x/100$ ---- $(x+1 - ((x+1) - (x/100))/100$ ----
 ---- $((((x+1)+1)+1) - (((x+1)+1) - (x+1)/100) - (x/100))/100$ ----

or

$x - x/100$ ---- $x+1 - ((x+1)-(x/100))/100$ ----
 ---- $[(x+3) - ((x+2) - (x+1)/100 - (x/100))/100]/100$
 where $x=0$

This is the series. But is it possible to put this into a form that we can use to solve for x at a certain time? The reason our single integral doesn't work is just because the nature of this series. Salt is being added to the tank after each interval. After that a certain amount of salt remains in the tank. This salt has to be accounted for.

Let's examine the series more closely.

$x - x/100$ ---- $(x+1 - ((x+1)-(x/100))/100$ ----
 ---- $[(x+3) - ((x+2) - (x+1)/100 - (x/100))/100]/100$
 where $x=0$

so x is 0 then

$(3 - [(2 - 1/100)/100])/100$

or t = time along the mixture of salt

$(t) - [(t-1) - (t-2)/100 + (t-3)/100.....(t-(t-1))/100 + (t-t)/100]/100$

taking into account that both sides the salt in the tank and the salt leaving the tank

Let $x = t - [(t-1)/100 + (t-2)/100 + (t-3)/100 (t-(t-1))/100....(t-t)/100]$

so

$s(x) = x - x/100$
 where $x = t - [(t-1)/100 + (t-2)/100 + (t-3)/100 (t-(t-1))/100....(t-t)/100]$

restated:

$t - [(t-1)/100 + (t-2)/100 + (t-3)/100 (t-(t-1))/100....(t-t)/100]$
 minus
 - $[t - [(t-1)/100 + (t-2)/100 + (t-3)/100 (t-(t-1))/100....(t-t)/100]]/100$

so building from this series:

$t - [\text{the summation of the series to time } t] - [\text{salt at that instance of } t]$
restated

$$[t - \int_0^{t-1} (t-y)/100 \, dy] - [t - \int_0^{t-1} (t-y)/100 \, dy]/100$$

$$t - [(y*t - (y^2)/2)/100] - [t - [(y*t - (y^2)/2)/100]]/100$$

Warning!!! This equation does not work. We have to fix it. ;)

Yes, this equation does not hold true for higher values of t . But I think it is a step towards finding something different. So get on the message board and leave some feedback and maybe we can solve this problem and apply that knowledge towards other mixture problems. Perhaps you will see something I missed.

This math problem isn't complete yet.

Billard Ball

16-Apr-1995

Unsolved Problem 16:

Does every obtuse triangle admit a periodic orbit for the path of a billiard ball?

We assume that the billiard ball bounces off each side so that the angle of incidence equals the angle of reflection. If it hits a vertex, it rebounds along the reflection of its entry path in the angle bisector of the angle at that vertex. The orbit (or trajectory) is periodic, if after a finite number of reflections, it returns to its starting point.

Reference:

[Croft 1991]

Hallard T. Croft, Kenneth J. Falconer, and Richard K. Guy, Unsolved Problems in Geometry. Springer-Verlag. New York: 1991. Page 16.

Revision 1 posted 4/25/95.

Each week, for your edification, we publish a well-known unsolved mathematics problem. These postings are intended to inform you of some of the difficult, yet interesting, problems that mathematicians are investigating. We do not suggest that you tackle these problems, since mathematicians have been unsuccessfully working on these problems for many years.

Problem excerpt from: <http://cage.rug.ac.be/~hvernaev/problems/Proble16.html>

Well is an unsolved math problem a challenge to solve or is it not worth trying to solve. You, the reader, be the judge. Even if it is impossible to solve, we, the mathematicians, will have fun trying. And who knows what solutions or techniques will be learned along the way.

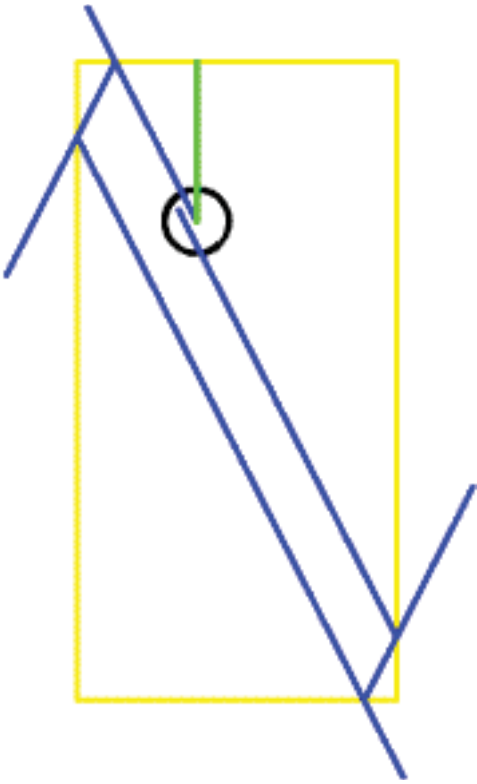
This billiard ball problem question doesn't explain it self thoroughly. My approach is an attempt to find a pattern. Finding a series or way to add the summation of numbers when the ball returns to its start point. This pattern may not occur in the first bounces, but how the ball progresses over time.

To do this I have attempted to write a program that would let us see where the ball lands after each bounce and give the coordinates at which this occurs. Here I have finally found a use for the ssine and scosine. Learn more about them here.

I take either the ssin or scos depending upon the angle and use it to determine where the ball will hit the table given the angle at which it is hit. Since the size of the table is constant, the length along the axis to one side of the table can be measured. Once the ball strikes the table, the distance to the other side equals the width or length of the table.

For example, if the ball is on a four foot by eight foot table the x-axis distance is 4ft. And since that perpendicular distance from the point is known as 4ft we apply the scosine. We take the angle the ball hits the side and determine what length at the given angle it will take to reach 4ft. This length is the scosine. And as long as it doesn't hit the y-axis rail, we can use it to find the position of the second place the ball hits the rail.

This is were the problem begins. I put the computer code in here in its text form. I haven't put it in a syntax of some computer language yet, but the logic is there. It reads in order from top to bottom. It also has some nested loops that can be confusing. I added comments were I could. I have included the text so that more people can improve upon it. Hopefully I will be able to convert it into C++ or Matlab syntax later.



```

given_angle = 117;
table_x_length = 4;
                table_y_length = 8;           // the given, that must be input
given_x_length = 3.1;
given_y_length = 6.2

reference_length = 8 - given_y_length           //that is using angles < = 180; forward hit

start_length =(given_x_length^2 + given_y_length^2);

                count = 2;           // count = 2 to count the times the ball has moved
                from the point it first hits the table's rail


                if cos(given_angle) > 0 and cos(given_angle) < = 1           //finding the reference angle
                to compare the given //angle
cos_reference_angle = (180)
else
cos_reference_angle = (0);           // cos(given_angle) > -1 and < 0

if sin(given_angle) < 0
sin_reference_angle = (270)
                else
sin_reference_angle = (90)           //else if sin(given_angle) > = 0


ssin = (sin(sin_reference_angle) - sin(given_angle))/sin(given_angle) * reference_length

(reference_length + ssin) * cos(given_angle) = x_length

if x_length > (table_x_size - given_x_length) //this determines if it hits the
                //x side or side rails

if given_angle < 90
reference_length = table_x_length - given_x_length
else
reference_length = given_x_length

                scos = (cos(cos_reference_angle) - cos(given_angle))/cos(given_angle) * //finds values
reference_length           //of first x hit

x_length = (reference_length + scos) * cos(given_angle)
y_length = (reference_length + scos) * sin(given_angle)

new_angle = given_angle
                new_angle = new_angle + (cos_reference_angle - given_angle) * 2
//this is the new angle formed by the angles reflection

```

```

use = 2 //use to determine angle since angle changes
//depending on which side was hit first
else //here x is hit first
if given_angle < 180 //if hits y rail first find y
sin_reference_length = 8 - given_y_length //values
else
sin_reference_length = given_y_length

x_length = (reference_length + ssin) * cos(given_angle)
y_length = (reference_length + ssin) * sin(given_angle)

new_angle = given_angle
new_angle = new_angle + (cos_reference_angle - given_angle) * 2

use = 1 //use to determine hitting y first

If use = 1 //this calculates if it hit the y distance of table first
ssin = (sin(sin_reference_angle) - sin(given_angle)) / sin(given_angle) * reference_length
if y_length + ssin * cos(given_angle) > table_x_length
scos = (cos(cos_reference_angle) - cos(given_angle)) / cos(given_angle) *
table_x_length
x_length002 = (table_x_length + scos) * cos(given_angle)
y_length002 = (table_x_length + scos) * sin(given_angle)
new_angle = new_angle + (90 - (cos_reference_angle - given_angle)) * 2
else
x_length002 = (table_y_length + ssin) * cos(given_angle)
y_length002 = (table_y_length + ssin) * sin(given_angle)
new_angle = new_angle + (cos_reference_angle - given_angle) * 2

If use = 2 //this calculates if it hit the x distance of table first
scos = (cos(cos_reference_angle) - cos(given_angle)) / sin(given_angle) * reference_length
if x_length + scos * sin(given_angle) > table_y_length
ssin = (sin(sin_reference_angle) - sin(given_angle)) / sin(given_angle) *
table_y_length
x_length002 = (table_y_length + scos) * cos(given_angle)
y_length002 = (table_y_length + scos) * sin(given_angle)
new_angle = new_angle + (90 - (cos_reference_angle - given_angle)) * 2
else
x_length002 = (table_y_length + ssin) * cos(given_angle)
y_length002 = (table_y_length + ssin) * sin(given_angle)
new_angle = new_angle + (cos_reference_angle - given_angle) * 2

```

While ((y_length - given_y_length) / (x_length - given_x_length)) is not equal to
((y_length - y_length002) / (x_length - x_length002))

```

        If use = 1      //this calculates if it hit the y distance of table first
ssin = (sin(sin_reference_angle) - sin(given_angle))/ sin(given_angle) * reference_length

if y_length + ssin * cos(given_angle) > table_x_length

        scos = (cos(cos_reference_angle) - cos(given_angle))/cos(given_angle) *
        table_x_length

x_length003 = (table_x_length + scos) * cos(given_angle)
y_length003 = (table_x_length + scos) * sin(given_angle)

new_angle = new_angle + (90- (cos_reference_angle - given_angle)) * 2)

else

x_length003 = (table_y_length + ssin) * cos(given_angle)
y_length003 = (table_y_length + ssin) * sin(given_angle)

                                new_angle = new_angle + (cos_reference_angle - given_angle) * 2


        If use = 2      //this calculates if it hit the x distance of table first
        scos = (cos(cos_reference_angle) - cos(given_angle))/ sin(given_angle) *
        reference_length

if x_length + scos * sin(given_angle) > table_y_length

        ssin = (sin(sin_reference_angle) - sin(given_angle))/sin(given_angle) *
        table_y_length

x_length003 = (table_y_length + scos) * cos(given_angle)
y_length003 = (table_y_length + scos) * sin(given_angle)

new_angle = new_angle + (90- (cos_reference_angle - given_angle)) * 2)
else

x_length003 = (table_y_length + ssin) * cos(given_angle)
y_length003 = (table_y_length + ssin) * sin(given_angle)

                                new_angle = new_angle + (cos_reference_angle - given_angle) * 2


x_length = x_length002
y_length = y_length002
x_length002 = x_length003
y_length002 = y_length003
    
```

References:

Learning MatLab 6.5, 1984-2002 The MathWorks, Inc.

Analytic Trigonometry with Applications: Sixth Edition, Barnett - Ziegler 1995 PWS Publishing Company

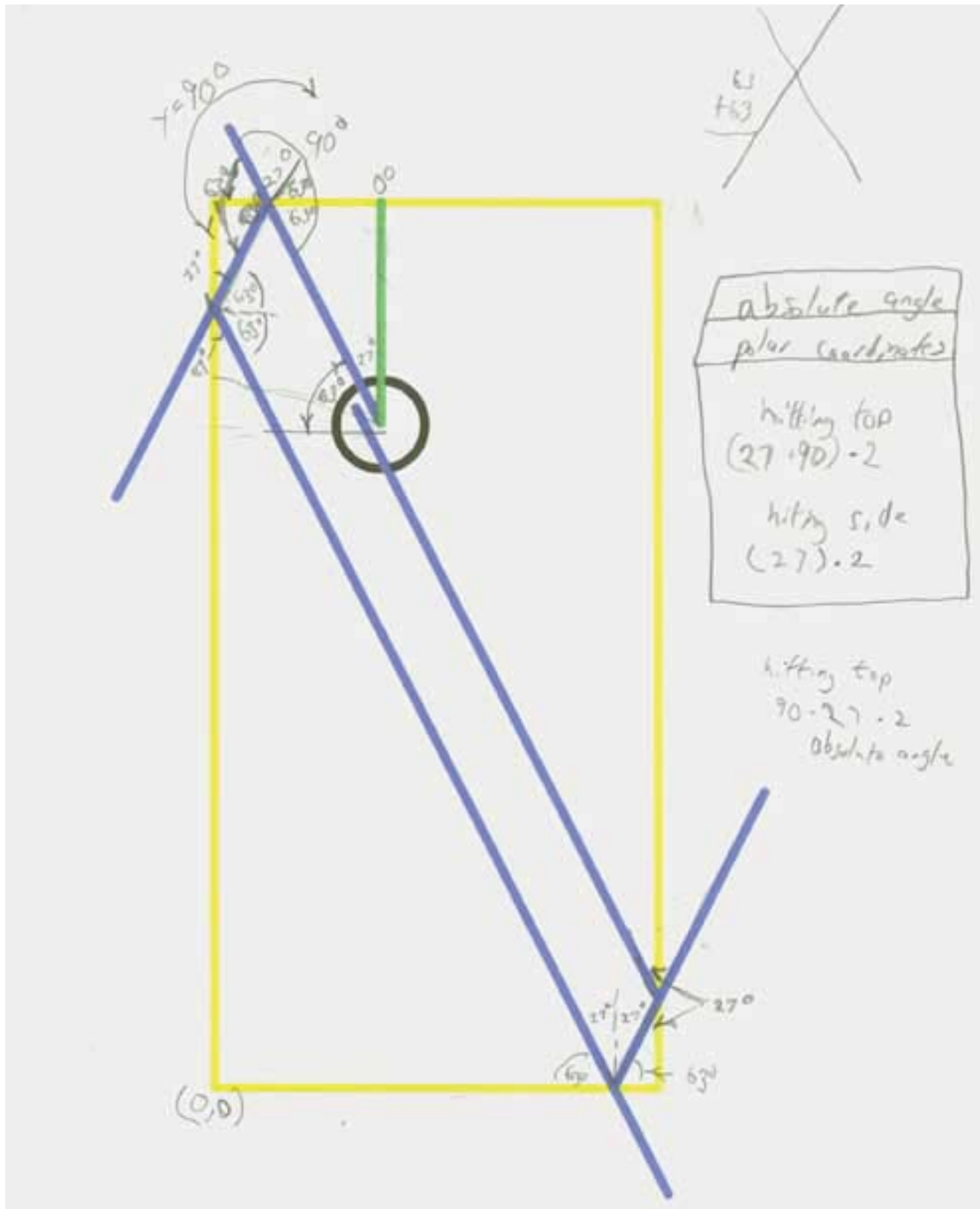
03-17-05

Billard Ball Problem

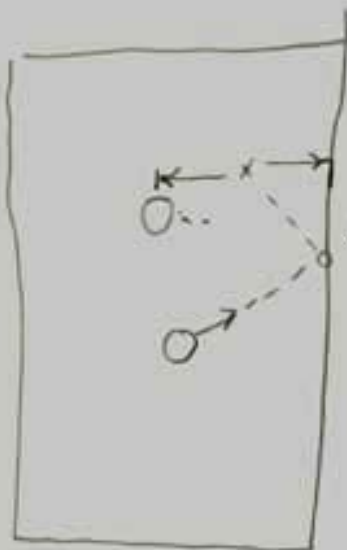


- use \sin & \cos to determine length and place where ball hits wall
- use absolute angles or polar coordinates to determine direction
- Find series of the periodic motion
- use slope of point at wall and point started and compare it to the start point vs second point where ball hits wall

Computer ALGORITHM



scratch

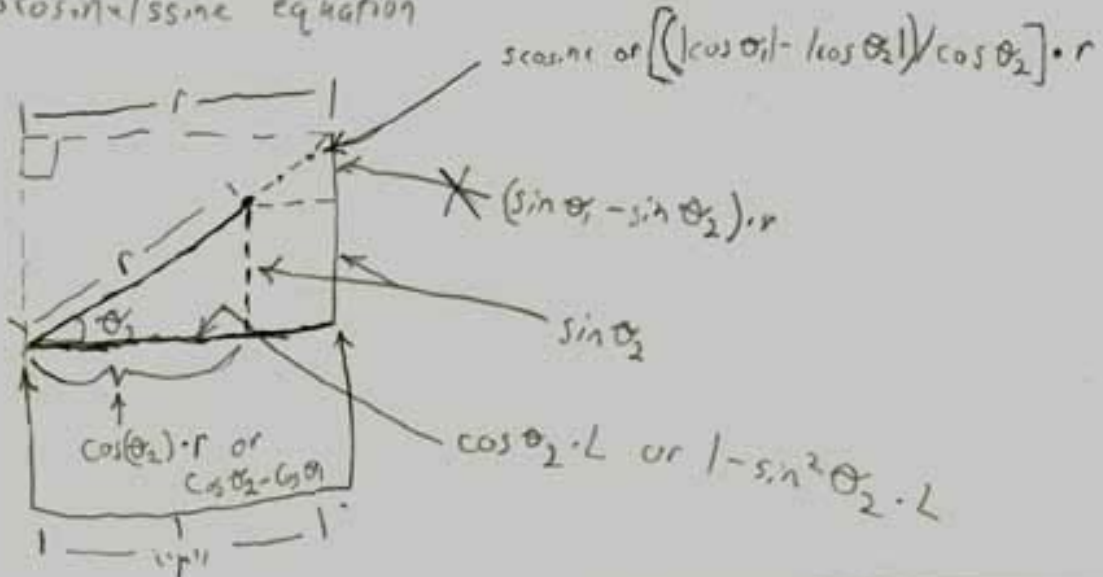


Can still use cosine even if angle is too hard to factor in

cannot exceed distance of start point
in other words, the maximum x-distance equals that of the distance at the start point

angle $+90^\circ$

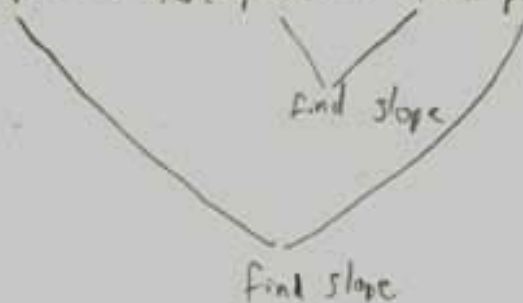
If distance $> x$ then use distance x as the distance in the cosine/sine equation



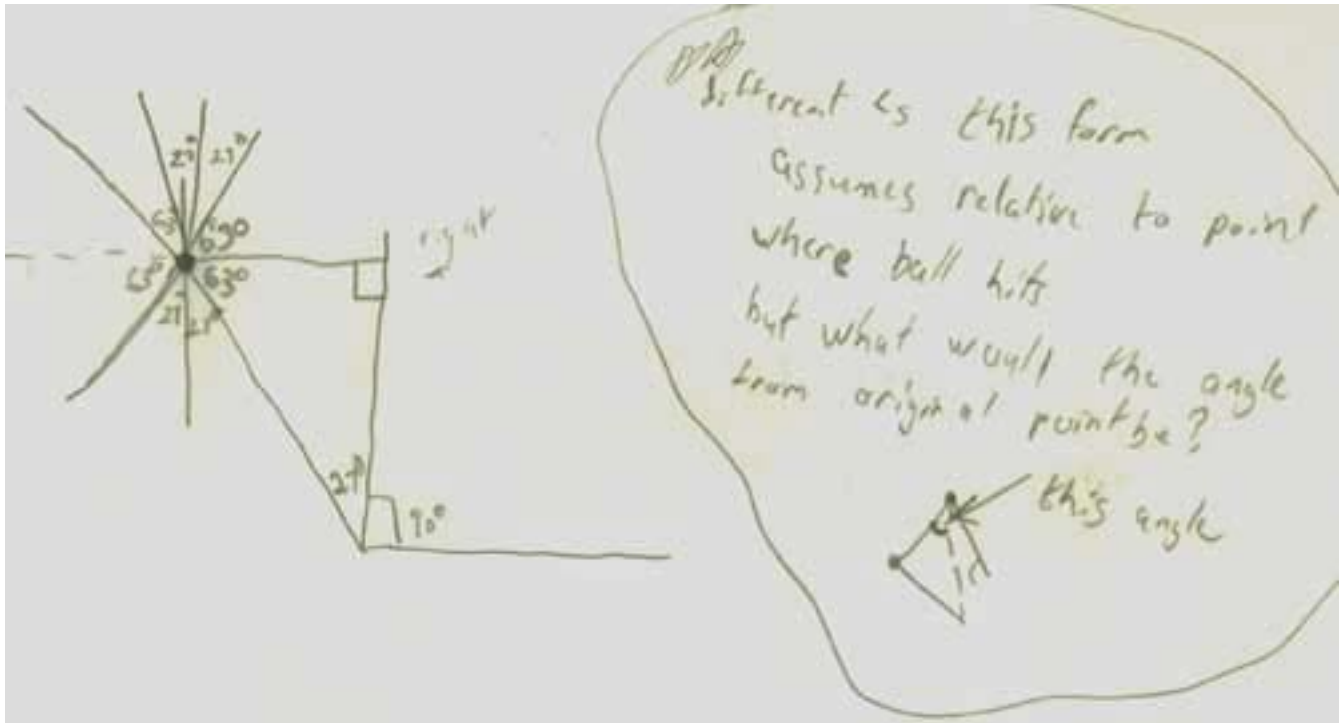
test
for
periodic

0, 0 + new x, new y + relative x, relative y + ... starting point

Start point ... next point ... next point



if slopes are = then periodic

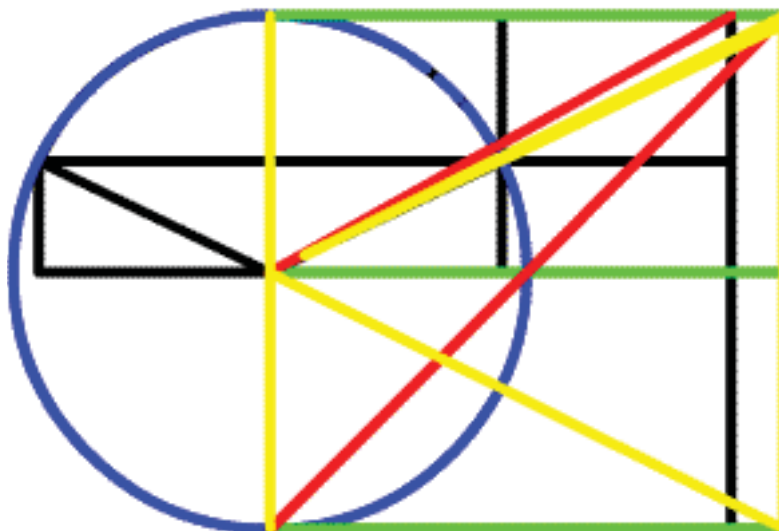


Circle Gets a Square

A circle is usually described by its radius or arc length or rather its circumference. But what if someone took the given dimensions of a circle and describe it by a square that the circle is inscribed within?

That is correct a square describes, if not defines, a circle. And that is what the challenge is:

Can a square's sides where each 4 sides equals the radius of the circle, be found thus determining the radius and diameter of the circle when the given information is only partially given? Specifically if the measurements are given from a circular arc with the cord and length the ends of the cord. (See the arc doorway problem. This is a twist to the arc doorway problem, but the method, if discovered, would apply to many applications and a new way to describe circles and squares.)



Application of the Problem:

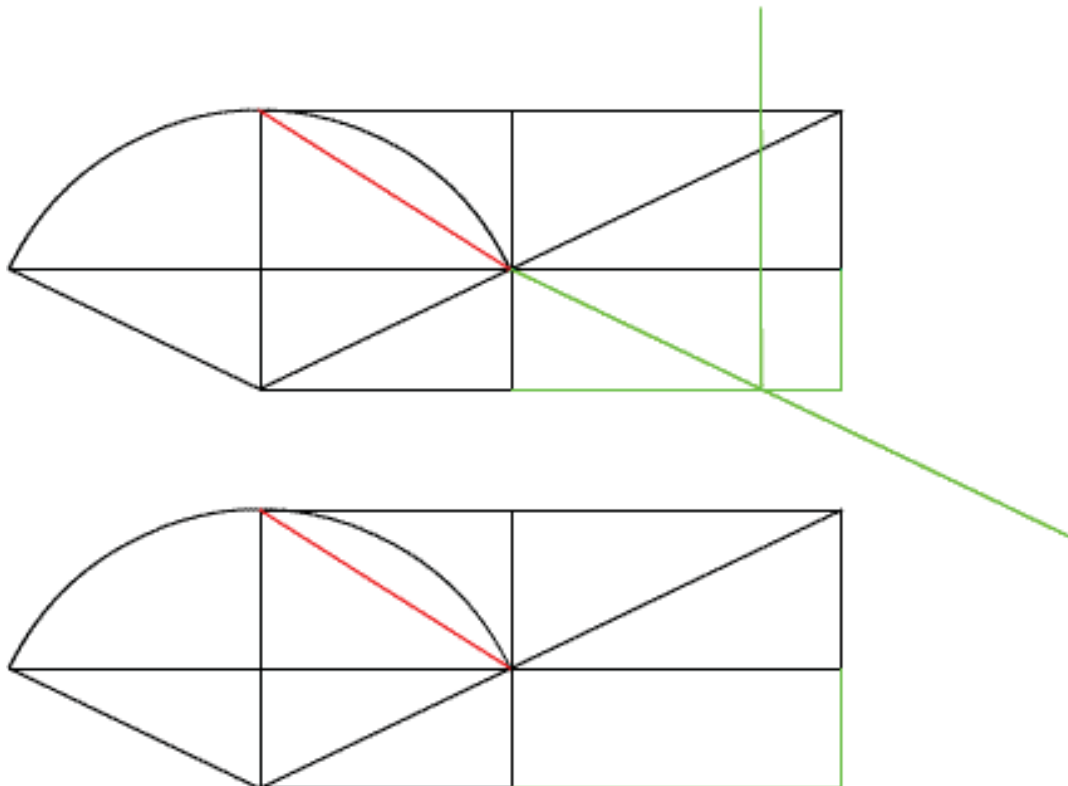
The arc door problem is used here because there is already two methods (Newton's Method and the geometry of the arc combined with the Pythagorean Theorem) by which it can be solved. Also if you can solve the square it will lead to a greater use of determining the curve on graphs. Specifically, finding out if a curve graphed $f(x)$ is circular. Since all arcs are sums of circular arcs, you should be able to describe parts of the graph and how much the arc is from one of the circular curves. If the square could be solved it would be excellent for curve fitting computer software programs.

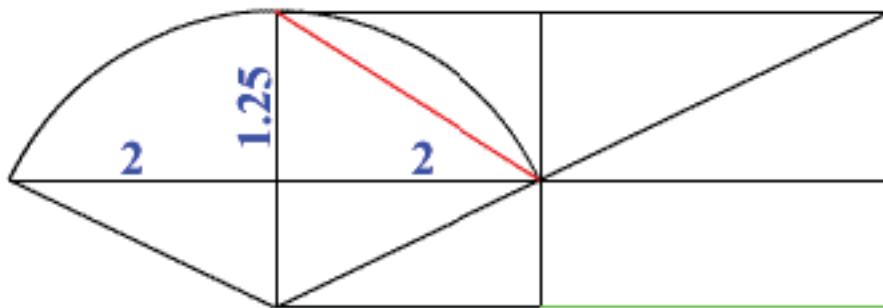
This is no easy task and it may be impossible without the help of higher mathematics, but I believe this problem is possible even though there doesn't seem to be enough measurements given. But it is necessary to know what solving this problem means. It would give a way to find a circle with limited measurements, but that is not the only use. As described in the last paragraph it would be a curve fitting formula with simple geometry. And possibly it would grant the ability to determine how far a curve is from being a circle by comparing the rectangle created to the square that would be formed by a circular arc.

Presentation of Problem:

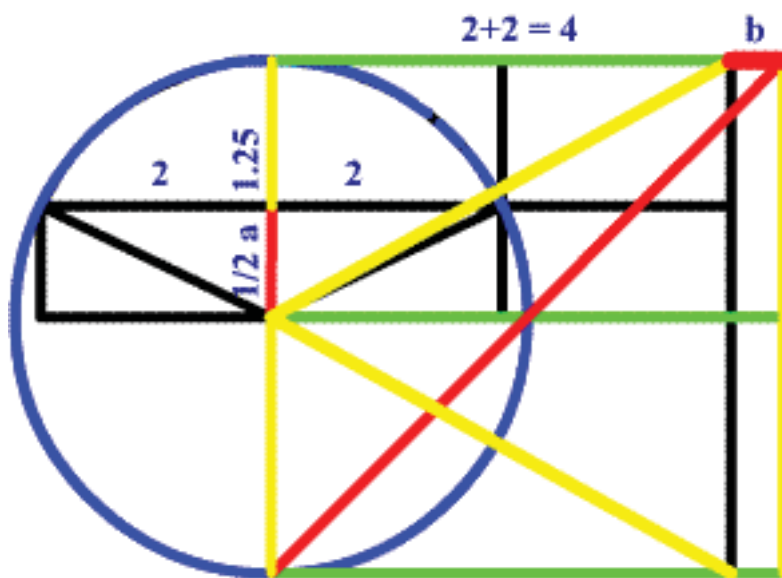
Remember this problem is without a solution and it is useful to use a circle with dimensions already know. Also note use more than one circle to make sure your assumptions work for all circles and arc lengths.

Here is the problem:





As seen here there is a circle with limited dimensions. This is a circle arc and has methods to solve it, but here notice that this circle is determined by a square. So the problem solver comes to the conclusion that if the square can be solved so can the circle. Plus it has the benefits of other solutions such as determining if an arc is circular as mentioned earlier. Now lets us look at what we can extrude from the given.



Note red line is at 45 deg. angle

Yellow is 30 deg.

But note these lengths change with different arc and circle sizes.

Here the radius is $(\frac{1}{2} a + 1.25)$ on the vertical length of the square and $(\frac{1}{2} b + 2)$ on the horizontal. The diameter is $(a + 2.5)$ on the vertical length and $(b+4)$ on the horizontal.

So that:

$$(a + 2.5) = (b + 4)$$

We will use these lengths as reference, but we are not limited to them. A better division of the square may be found latter. (Note if the values of a and b are substituted it just leads to a $0 = 0$ equation with infinite values. You will see latter.)

With this given. Determining the radius or square seems like it should be possible, but it just seems we do not have enough values to work with. We can create equations that equal each other, but they just yield infinite many solutions. What we must find is an equation that describes the value of a or b in which a solution of a or b can be found. We need an equation that gives a value of a or b that doesn't include it with a variable of the radius or other part of the square. Or we must find the value of the radius. This seems like it should be able to be done by simple algebra, but the calculation is more complex than thought on first inspection.

Let's start with what is known about a square. (It will help to have a compass handy to look geometric constructions.) A square is separated into 2 equal triangles with a 45 deg. angle. This angles cosine and sine are equal. (Both equal the radius of the circle.) So if this 45 deg. angle is drawn from the bottom left to the upper right of the square it would mark the diameter of the circle and show where the square's upper right side ends.

Also in a square if a 30 degree angle is drawing from the center to the upper right, its aligned length is equal to the radius. Also it's sine is equal to $\frac{1}{2}$, so the y distance from the center equals $\frac{1}{2}$ the radius.

Some equations are:

$$(\cos 30 * r)^2 + (\frac{1}{2} * r)^2 = (r)^2$$

and

$$\cos 45 * (\sqrt{r^2 + r^2}) = r$$

These equations are derived from the given. There are more variations of them, but by themselves they don't lead to an answer without variables.

Now let's look at this angle relationship of the 30 deg. and 45 deg. angle and see how it can be applied to solve the radius (a + 1.25) or ($\frac{1}{2}$ b + 2). But my last attempt has to do with were the 30 deg. angle and 45 deg. angle intersect. Normally these angles will not intersect, but the 45 deg. angle is placed here $\frac{1}{2}$ the radius below the 30 deg. angle. But unfortunately the answer is still in the form of the radius.

$$r - (r * \cos 30) = 0.1340 * r$$

The length of 0.1340 is proportional to all circles. That length of 0.1340 is the length between 30 degrees and 45 degrees on a unit circle.

In Conclusion:

This one is a stinker. It seems like it is possible to solve on a hunch, but there are so many unknowns. That combined with the fact that most of the equations still rely on the radius is making it difficult to use the equations. Still I think it is only a problem to tinker with when you, the mathematician are bored. There may be something here, but after hours at looking at this problem, I haven't found it. This is a problem you tinker with in your free time. But don't forget we already have 2 solutions for finding the radius of the circle without the square. I just want to find a way to solve the square.

Circle Key

A few things to note before the problem is presented:

Even though the parabola of a second degree polynomial equation may lie in negative (or imaginary) numbers, the parabola itself may still be measured using relative coordinates. In other words no matter where the parabola lies on the coordinate plane, the measured values of the parabola itself are still useful.

The goal of this problem is to find a circular that encompasses a segment (and a line perpendicular to that segment). (Yes, it is the "arched doorway problem revisited".) But what would be an application of this problem? It is another way of solving circular arcs and may lead to a better way to describe curves. But what I am looking for is an alternative to the Quadratic Equation. Finding a alternative to the Quadratic equation may seem too difficult, but this problem is a place to start. Another application of knowing what radius will encompass what line segment is a possible method of encryption. Matrices are often used for encryption. Matrices explain linear equations. The question is what if we had a way to better describe polynomial equations?

$$\left[\sin 30^\circ \cdot \sqrt{\frac{1}{2} (b+4)^2 + (b+4)^2} \right] = \left[\frac{1}{2} (b+4) \right]$$

$$\left[\frac{1}{2} (b+4) \right]^2 + \left[(b+4)^2 \right] = \left[\frac{\frac{1}{2} (b+4)}{\sin 30^\circ} \right]^2$$

$$\left(\frac{1}{4} b^2 + 2b + 4 \right) + (b^2 + 8b + 16) = \frac{\frac{1}{4} b^2 + 2b + 4}{\sin^2 30^\circ}$$

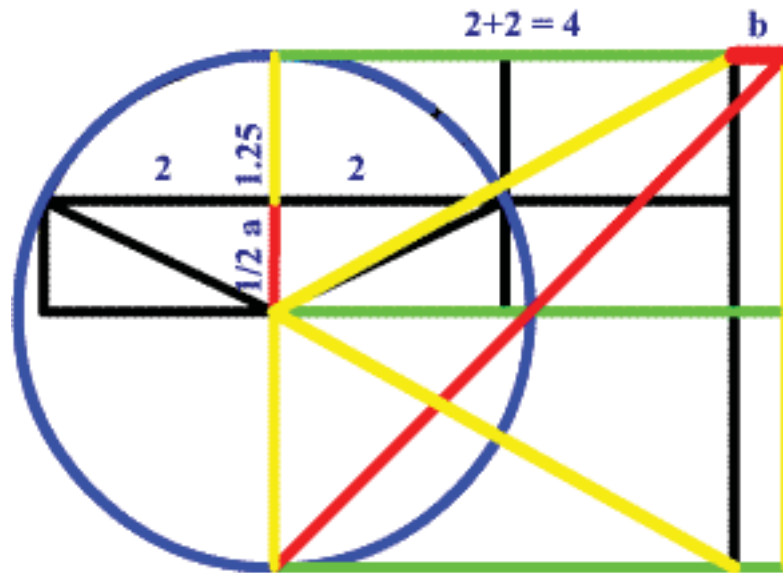
$$\frac{1}{4} b^2 + 10b + 20 = b^2 + 8b + 16$$

$$\frac{1}{4} b^2 + 2b + 4 = 0$$

$$4 \left(\frac{1}{4} b^2 + 2b + 4 \right) = 4(0)$$

$$x^2 + 8x + 16 = 0$$

Now let's explore where these equations come from. Then we will see what they can do.



Note red line is at 45 deg. angle

Yellow is 30 deg.

But note these lengths change with different arc and circle sizes.

The equation originates as a square above and inscribed circle. The length of the square is equal to the radius of a circle. Here $b + 4$ equals $a + 2.5$. We could try solving the sides by substitution but there is no easy (if any) way to isolate one of the variables (a or b). So that is the problem. We have to find at least 2 ways to describe b (or possibly a) so that we can get a real number value for b from those equations. And we will do it using elementary mathematics.

Well a square is a special polygon where the length equals the width. So in this example $b + 4$ is equal to $a + 2.5$. So relying on basic understanding of the Pythagorean Theorem there is a hunch that it is possible to solve for a or b . So $\text{Sqrt}((a + 2.5)^2 + (b + 4)^2) = \text{unknown}$. So the equation appears to go nowhere. But if since $(a + 2.5)$ equals $(b + 4)$ we can substitute the values. So $(b + 4)^2 + (b + 4)^2 = \text{unknown}$.

So in order to find another equation that describes b in a different way (a different way to achieve the same value), we must rely on the only other thing at our inventory. That is the trigonometric functions: sine, cosine, tangent, etc.

It helps if we use our known special angles: 30, 45, 60 degrees respectively. If we were to draw a 30 degree angle from half the length of the length, this 30 degree angle would end at the top of the square since the $\sin 30 = 0.5$. Now we have just the observations we need to make use of the Pythagorean Theorem. The above equations could have been simplified in less steps, but the $\sin 30$ shows where the values come from. The equation is actually very simple: $b^2 + 8b + 16 = 0$

$$y = x^2 + 8x + 16$$

So we have the equation what we do with it. We have to solve for the parabola's x and y values by using the Quadratic Equation.

For example the example we have been using (the "Arch Doorway" problem), we would use the given length of the segment (the y value) and just plug it into the equation.

For an easy starting point you could just plot the graph of the parabola. Then you take the y value as the value of the line segment (1.25 in the arch door ex.) and measure the x length span of the parabola. The radius of the desired circle equals the space in between the sides of the parabola. So the difference of the second x value minus the left x value equals the radius. This is the radius of the circle that would encompass the segment.

Finding the roots of quadratic equation: Quadratic formula

If $ax^2 + bx + c = 0$, then the solutions (roots/zeros) are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (\text{this is called the Quadratic formula}).$$

the following graphics were produced using a quadratic calculator found at:

<http://www.hostsrv.com>

$$y = x^2 + 8x + 16$$

$$X^2 + 8X + 16 = 0$$

plug in value

$$X^2 + 8X + 16 = 1.25$$

$$X^2 + 8X + 14.750 = 0$$

$$X = \frac{-8 \pm \sqrt{5}}{1}$$

$$X = -5.1180$$

$$X = -2.8819$$

$$-2.8819 - (-5.1180) = 2.2361$$

$$\text{radius} = 2.2361$$

$$y = x^2 + 8x + 16$$

Result

Before plotting the parabola, we need to find a number of things which will help us figure out what the graph should look like :

Concavity [[explain](#)]

The parabola opens upwards.

x-intercepts [[explain](#)]

$$x = -4$$

y-intercept [[explain](#)]

$$y = 16$$

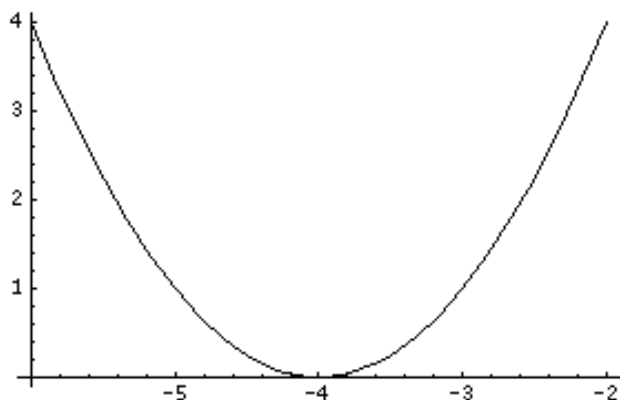
Axis of Symmetry [[explain](#)]

$$x = -4$$

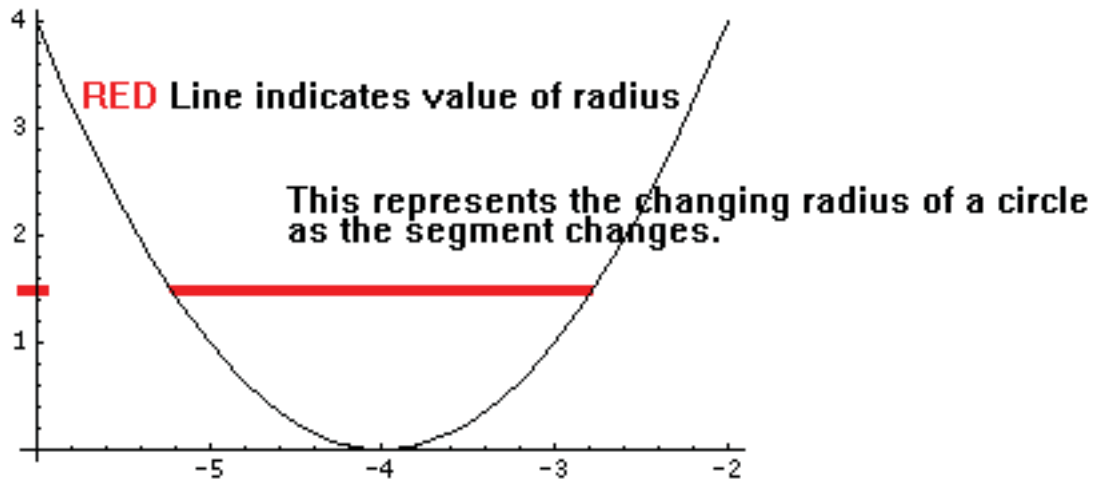
Vertex [[explain](#)]

$$(-4, 0)$$

Putting all these clues together, this is what the graph looks like :



NOTE : The intersection of the axes in this plot may not occur at (0,0)



So from here as long as we have some understanding of why the equation and process works it is simply a matter of substituting values. We can shift the position of the parabola to make the math easier. We can also make it so that the value of radius we need occurs at the x intercepts. As demonstrated in the following examples.

$$y = x^2 + 8x + 14.75$$

Result

Before plotting the parabola, we need to find a number of things which will help us figure out what the graph should look like :

Concavity [[explain](#)]

The parabola opens upwards.

x-intercepts [[explain](#)]

$x = -2.88197$, $x = -5.11803$

y-intercept [[explain](#)]

$y = 14.75$

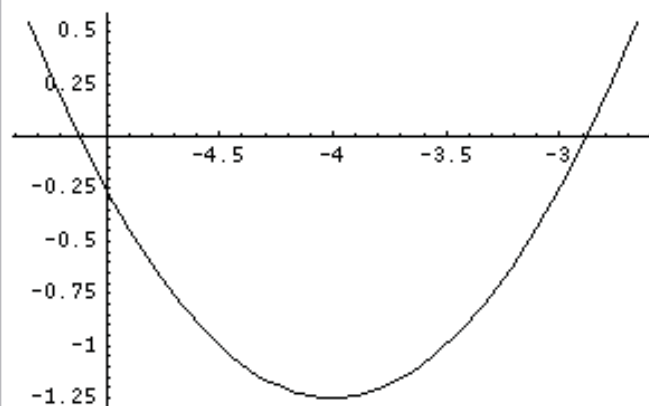
Axis of Symmetry [[explain](#)]

$x = -4$

Vertex [[explain](#)]

$(-4, -1.25)$

Putting all these clues together, this is what the graph



NOTE : The intersection of the axes in this plot may not occur at (0,0)

$$y = x^2 + 8x + 11$$

Result

Before plotting the parabola, we need to find a number of things which will help us figure out what the graph should look like :

Concavity [[explain](#)]

The parabola opens upwards.

x-intercepts [[explain](#)]

$x = -1.76393$, $x = -6.23607$

y-intercept [[explain](#)]

$y = 11$

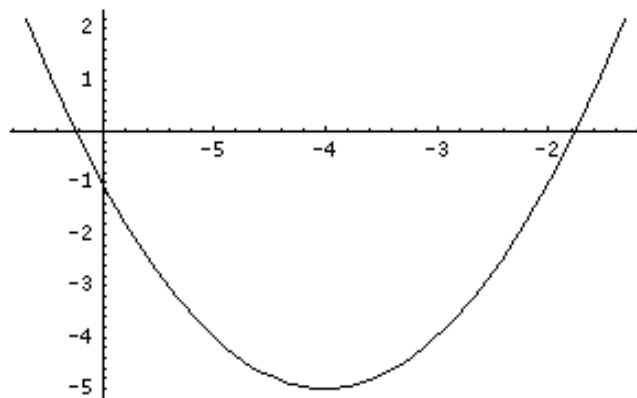
Axis of Symmetry [[explain](#)]

$x = -4$

Vertex [[explain](#)]

$(-4, -5)$

Putting all these clues together, this is what the graph looks like :



NOTE : The intersection of the axes in this plot may not occur at (0,0)

The research into this problem is only beginning. This is just a rough draft it needs some tweaking. Be sure to check out the other math problem on the site. Many of them relate to this description. But until I finish documenting this problem... May the Creative Force be with You

Linear Parabola

The following pages of "Hunches" are presented for the possibility of creating new ideas. They are also interesting to see how the problem was approached. However, many things presented here though they may show creative thought are mathematically incorrect. Repeated these exercises are for ideas and contain many logic and mathematical mistakes. They are still interesting especially if the reader is looking for new problems to solve, getting ideas or brainstorming. That is why this section is recorded here.

20060103

On a circle 2 different angles will yield the same sine (y component value) if spaced a certain given distance, a length away from the origin. This is not new.

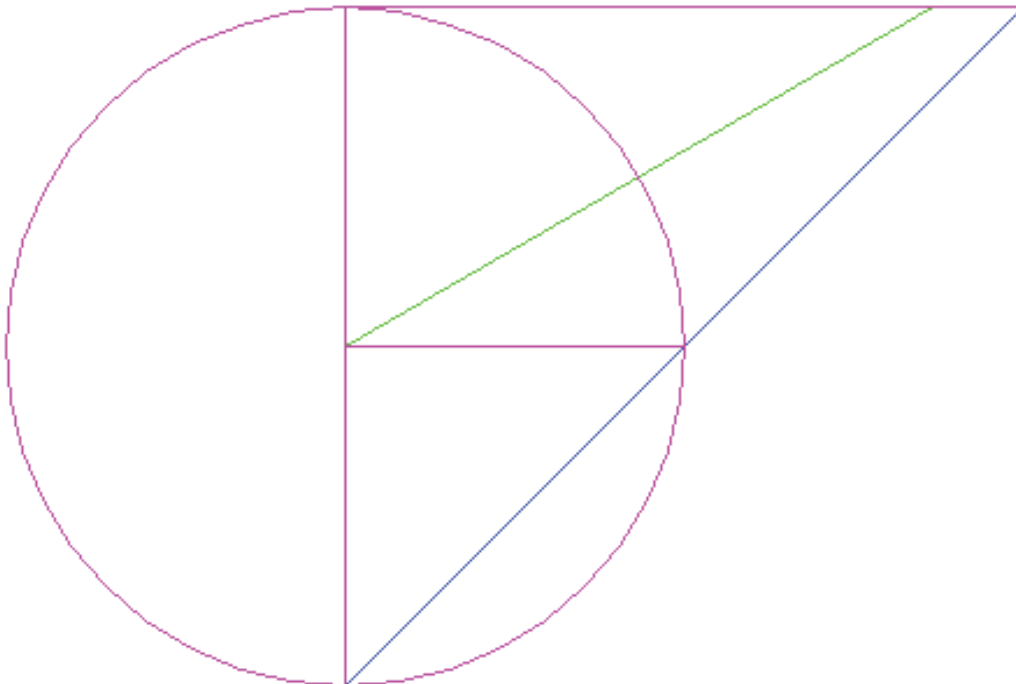
But for two angles 30 degrees and 45 degrees, their lengths may differ but if the 30 degree line is placed at the origin and the 45 degree angle is placed 1 radian or r away from the origin on the x-axis the lines will meet at a height of r (from the origin) and

$$\sin(45) * r = (\sin(30) * r) + r$$

It would be interesting to see what this looked like graphed on a unit circle. (Combine radians and degrees.)

$$\frac{1}{2} (\cos(30) * r) = \sin(45) * r$$

Need to find relationship (=) between angles.



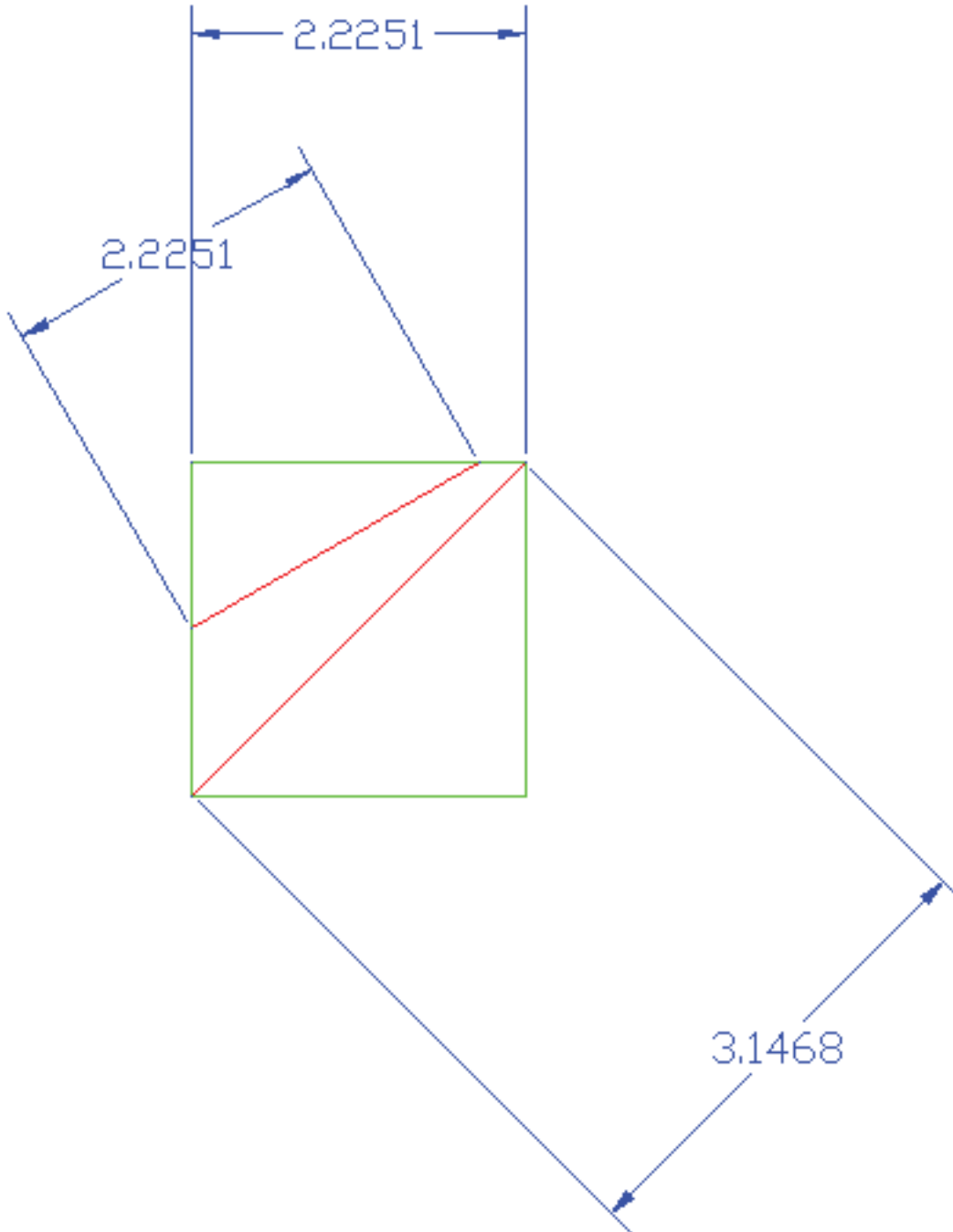
20060309

$(\text{Diameter} / \cos(45)) - r$

$r / \cos(45) - r = ???$

$(D * \sin(45)) - r = \sin(30) * r$

still can't describe it, but the idea is there



Let's look at a square the circumscribes the circle that we need values from.

In this example the square's length and width are 2.225125 half the size as the arched door problem, but the same applies to any square.

$$2.22512 * 2 = 4.45024 \quad r + r = D$$

$$4.45024 * \sin(45) = 3.14679488 \quad D * \sin 45 = r / \sin 45$$

is the length of the bisector of the square

$$3.14679488 - 2.225125 = 0.92167 \quad \text{bisector} - r = \text{difference of } 45 \text{ length} - 30 \text{ length}$$

$$3.1468 - 0.92167 = 2.225125 \quad \text{which is the length of the } 30 \text{ degree angle segment}$$

Need equation though

$$D * \sin(45) - (D * 45 - r) = \text{length of } r = \sin(30) * D$$

20060103

$$(\text{cord} / \sin(30)) + (\text{cord} / \sin(30)) + x = \text{Diameter}$$

Take a measurement from the mirror of the square at 30 degree angle. Will it reach the upper and lower corners of the square?

We are concerned with the top radius or top of the line segment. The length at which intersects the horizontal segment is the length of which the 30 degree angle goes beyond the horizontal segment.

Add this length to the original horizontal segment and divide by the $\cos(30)$. Divide this length by 2 and you have the radius.

It should work with any angle though it might require sin and cos angle to equal the radius such as a 45 degree angle. The $\cos(30)$ is longer than r and a 45 degree angle, but so is the square that mirrors 30 degrees.

Take an angle from the radius at 90 degrees. From the segment, determine what x direction (\cos) the angle crosses the segment. This should be the same length the bottom crosses the segment. (Hopefully) Take the \cos of that length and determine what \sin is needed to cross that length. Divide that length by the \sin .

****The preceding did not hold true. However it is on the right path****

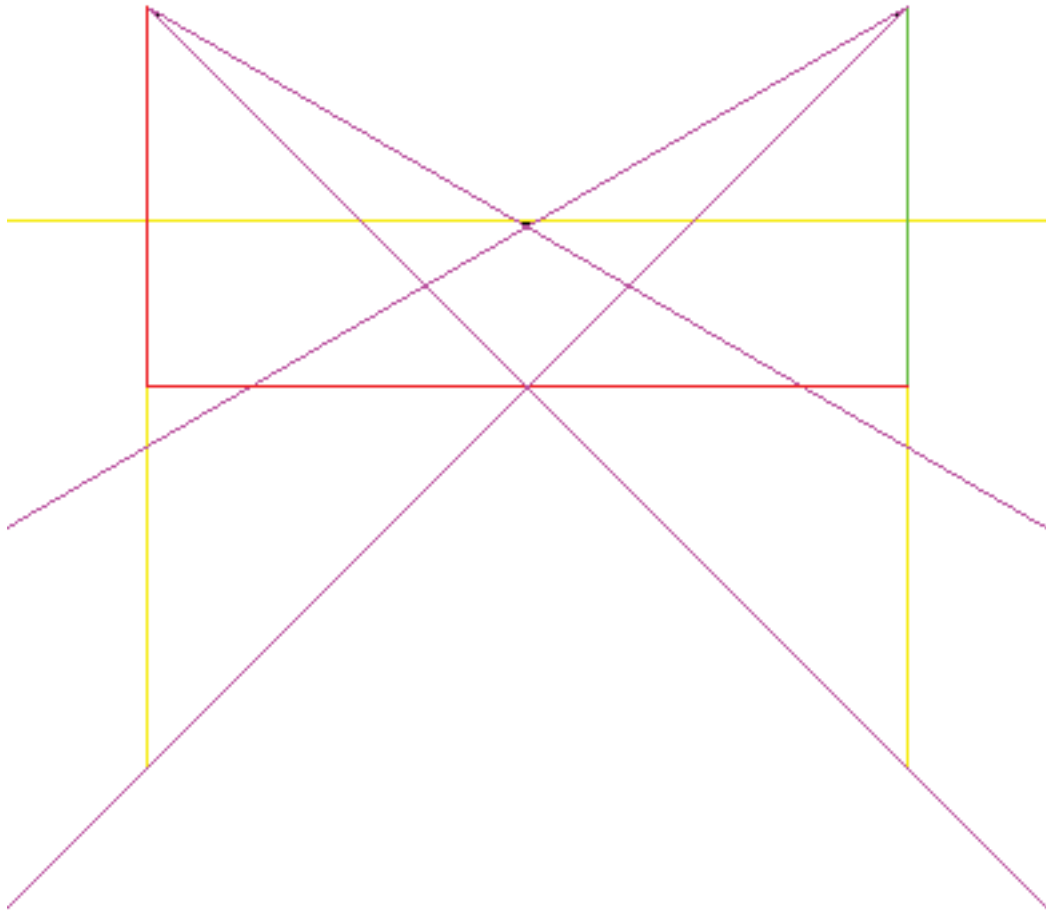
20060106

Even though without some new method the above wasn't true the ideas that form an equation and hold true is the following:

With the mirrored angle starting at the upper corners of the square that surrounds the circle of unknown radius, the angles all intersect the corresponding mirrored angle in the middle of the x axis of the circle. So many angles to choose from (special values), they can be found were $\frac{1}{2}$ the cosine below the given segment of the circle to have the same x value.

20060107

Why is solving this and finding an equation that describes it so valuable? This would mean we could find a circle only knowing 1 value. That value is the value of the segment.



20060313

(percentage of difference between 30 and 45 lengths) * sin(30) * cord = x + sin(45) * cord

$$[\sin(45) * \text{cord} + x] / [\sin(30) * \text{cord}] = \text{percentage difference}$$

recall

$3.14679488 - 2.225125 = 0.92167$ bisector - r = difference of 45 length - 30 length

test for r = 2.225125

$$[\sin(45) * \text{cord} + x] / [\sin(30) * \text{cord}] = 1.92167$$

since $\sin(45) * \text{cord} + x$ is 1.92167 x larger than $\sin(30) * \text{cord}$

$$x = (1.92167 * \sin(30) * \text{cord}) - (\sin(45) * \text{cord})$$

$$x = 0.31716$$

$$[x / \sin(30)] + [\text{cord} / \sin(30)] = [x / \sin(30)] + [1.25 / \sin(30)] = r$$

$$r = 0.6343 + 1.5858$$

$$= 2.22012$$

good way to estimate length of r given cord length

However needs to be tested.

Temporary Work

20060612

It is a little confusing what exactly is trying to be solved with the "parabola key." After all a chord can be encompassed by infinitely many circles. However what we wish to find is the circle with the correct proportions. And also, equally important is finding the first circle (smallest) that encompasses it.

20060621

Let's take a look back at how the parabola was found. It was found by basic algebra.

$$[\sin(30) * \sqrt{\frac{1}{2} (b+4)^2 + \frac{1}{2} (b+4)^2}] = [\frac{1}{2} (b+4)]$$

$$[\frac{1}{2} (b+4)]^2 + [(b+4)^2] = [\frac{\frac{1}{2} (b+4)}{\sin(30)}]^2$$

$$(\frac{1}{4} b^2 + 2b + 4) + (b^2 + 8b + 16) = \frac{\frac{1}{4} b^2 + 2b + 4}{\sin^2(30)}$$

$$1 \frac{1}{4} b^2 + 10b + 20 = b^2 + 8b + 16$$

$$\frac{1}{4}b^2 + 2b + 4 = 0$$

$$4\left(\frac{1}{4}b^2 + 2b + 4\right) = 4(0)$$

$$x^2 + 8x + 16 = 0$$

Hence

$$x^2 + 8x + 16 = 0$$

with chord = 1.25

$$x^2 + 8x + 16 = 1.25$$

$$x^2 + 8x + 14.75$$

Solve for x using Quadratic Equation

$$\frac{x = -b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{x = -8 \pm \sqrt{8^2 - 4*8*14.75}}{2}$$

$$x = -2.8819 \text{ or } x = -5.1180$$

$$+ 4$$

$$1.1180 \text{ or } 1.1803$$

$$\times 2$$

$$2.2360 \text{ or } -2.2360$$

Recall that in order to solve the “arched doorway” problem a variation of the Pythagorean Theorem was used. If this formula is used to test values of the parabola the values would be taken as the chord in the x-direction and the radius in the y-direction. This works for small values and also checks by the Pythagorean Theorem, but we know just by first observation that although both equations are satisfied, the values are too large.

This leads the observer to test values with the chord in the x-direction and radius with the y-value. (Note all values are relative to the parabolas origin which is at (-4,0). So the x value would be added by 4.)

Here is a random test value: a chord of 28 plugged into the equation

$$x^2 + 8x + 16 - 24 = 0$$
 Four is added since the parabola is shifted -4 units to the left

then using the Quadratic Equation to solve

$$\text{we get } 44 \text{ or } -39.1918$$

Here 44 is used and plugged into the Pythagorean Theorem used to solve the “arched doorway” problem

First the “arched doorway” problem for reference:

$$a^2 + b^2 = c^2$$

$$1.25^2 + 2^2 = r^2$$

$$r^2 - 1.25^2 - 1.25^2 + 1.5625 = r^2$$

$$r^2 - 2.5r + 5.5625 = r^2$$

$$-2.5r = -5.5628$$

$$r = 2.22512$$

for 44 radius and 28 chord

$$(44-28)^2 + X^2 = 44^2$$

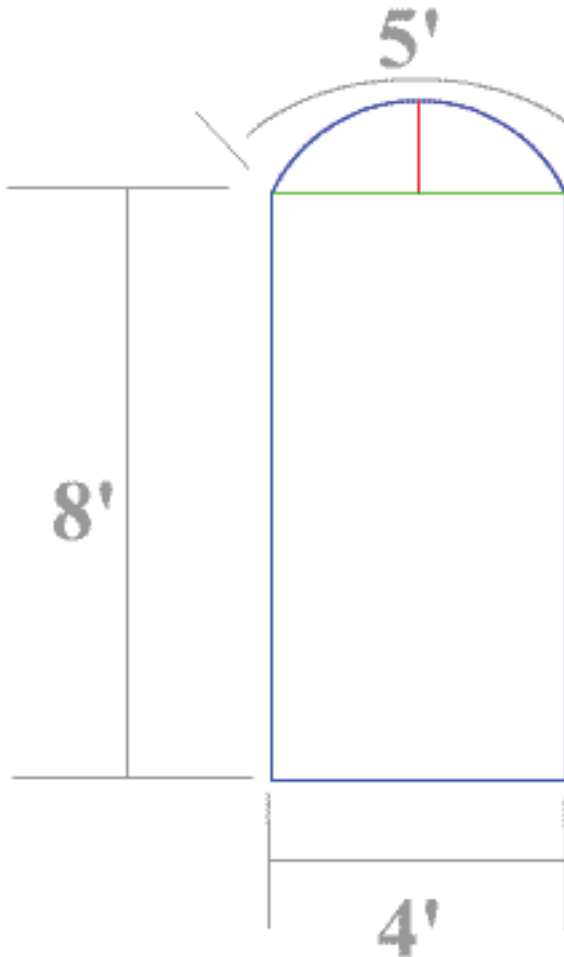
$$256 + X^2 = 1936$$

$$X = 40.9878$$

That is the value of the length perpendicular to the bottom of the chord

It checks. And it should be the smallest circle that encompasses a chord of length 28 units!

Chord Inverse



Temporary Work

20060514

From the Arched Door math problem from page 21 of Barnett and Ziegler

We are presented with a problem where the cord can be found ($4/5=1.25$). On Constructor's Corner we have already found 4 different ways to solve this problem. However this solution is simple and proves to be quite interesting.

Here is a simple, little theory that this solution is based on:

A chord will make a circle with a radius equal to the chord. But imagine if the chord increased in length as it rotated around the circle. This increase in length adding to the chord so that the chord now draws the circle that encompasses it.

It just so happens that the cord increases "inversely" to maintain the same arc length as the original radius.

In the Arched Doorway problem we already know that at 180 degrees along the horizontal line, the chord equals 2. (We would have to know an angle if the angle wasn't perpendicular.)

$$\text{So } 1/(1.25/2) = 2$$

the circle increases by so much of a percent per so many degrees

So the percentage increase doubles by 90 degrees; so the length of movement is 4 that is the horizontal distance 1 direction from 90 to 180 and then from 180 to 270

$$1/(1.25/4) = 3.2 + 1.25 = 4.45 \text{ This number is the diameter.}$$

$$(3.2 + 1.25)/2 = 2.225 \text{ This number is the radius.}$$

Chord Application

Temporary Work

20060119

$$\sin(30) * r = \frac{1}{2} * r$$

30 degrees and 30 degrees cross at center of square
so does any angle such as 40 and 60

but the length of $30 * r$ is less than r

How to tell when 2 lines cross?

Use Linear Algebra???

20060120

r in the form of a circle not a line must convert to a line for linear equation form. But shouldn't there be a matrixes form of polynomial???

20060210

Completing the square of a quadratic formula. That is exactly what the circle formed by the chord is. It is just another form of the quadratic equation. Have to test this.

$$f(x) = ax^2 + bx + c = f(x) = a(x-h)^2 + k$$

Since many equations have been found that describe a circle without using the radius and one of them uses the quadratic equation, maybe there is another simpler alternative to the quadratic equation. We know that there is in this instance.

20060212

Even though the parabola of a polynomial equation may lie in negative or imaginary numbers, the parabola itself may still be measured using relative coordinates.

Picture of parabola-key equation (handwritten for now)

20060509

So essentially the x distance from the vertex (the x distance is the length of the chord, which is smaller than the radius) will yield a "y" value on the parabola. This y-value is the radius.

Picture showing the parabola.

20060509

Three Hundred Forty Fourth Post: Math Application

Ok so we have the "Arched Doorway Problem" and the "Parabola Key" but other than finding a chord's encompassing circle and hence the radius is there any application? Is there an application that the contest on Constructor's Corner is based possible? And the answer is very simply yes. As the people work on entries to the contest, I continue to work on my own to create more interest.

In with alternating electric current there is a current and a voltage wave. It is described by a sine wave and measured in angles. But what if you only knew one value on both the current and voltage graphs (both occurring at the same time)?

Well with the "Parabola Key" (if it works) and the other methods for finding the radius of a chord, you would be able to find the radius that encompasses it. (The chord on a power graph will always be less than the radius.) Once you know this one radius the other parts of the sine graph can be determined because you know the time of when the chord occurred versus the difference of length from the radius.

Find the radius of both the voltage and current graph and determining the phase angle by the positioning of that radius, you now have the knowledge to determine the power. If you were to find the area of both the voltage and current by the formula $2\pi r^2$ (two pi r squared), you would have a complete description of the power so that voltage area + current area = new circle = total power. And the perimeter ($2\pi r$) around this power circle would describe the power at various instances. Much the same way as the hysteresis loop describes magnetic power.

So with 2 single chords we have determined the radius, phase angle, and power. We have also added the voltage and current graphs. This was done since the area under the sine curve is equal to the area of a circle with the same values.

I'm am still working on this but this is the main idea behind my work. It will be posted on both the Blog and message board.

So with new and interesting ideas... May the Creative Force be with You

Chord vs. Circular Function

Temporary Work

20060612 revised 20060712

So far explanations have been limited to using mostly algebra and trigonometry. However, an excellent way to use the parabola that is found by chord vs. radius length is to use some things learned in calculus.

So what is found by taking the derivative of a value on the parabola?

20060613

$$x^2 + 8x + 16 = 0$$

$$\frac{d}{dx} [x^2 + 8x + 16 = 0] =$$
$$= 2x + 8$$

20060614 — 20060705 revised 20060712

Ideally, if you could describe the circle over time you would have a new way to describe a parabola and ultimately the quadratic equation. (However this is based on finding a way to easily describe the circle.)

Calculus has many tools to describe functions and their curves. Perhaps a new math will be developed to explain curves using circles and how much the curve deviates from the circle. The new math would be called Circulus.

There is a problem with setting up a custom coordinate system or reference circle and have it describe the curve in a useful manner. It is definitely a challenge.

20060711 — 20060712

But before beginning a long mathematical journey to solve the unknown and uncharted curves, perhaps something simple has been overlooked. Sometimes in math all the power of calculators and complex mathematics such as calculus are not needed. Instead a simple algebraic formula does the job.

There is a simple discovery that might just prove to be the most useful application of the parabola that represents a chord. With that parabola, it is known what circle will encompass a given chord. But what if the chord lies on a circular function, such as the sine curve (or cosine) often used in the study of electricity? By knowing just the period, the cycle through which the graph makes, and the chord or value at a given x distance-- the maximum value of the circular function (also the radius of the circle) can be solved easily.

20060723 — 20060724

Attention: This is just a theory and an outlined experiment. It is not guaranteed to work. But sometimes the idea is just as interesting as a working solution.

Theory: If an arc that encompasses a chord on a sine curve (circular function) is known, the rest of the graph can be solved, with emphasis on the maximum value. We take the parabola, (Parabola Key), and find the positive value of the radius that corresponds to the given chord. It does not matter if this is the smallest encompassing circle. However, the graph must be of a circle with constant proportion. Proportions that are based on the reference circle around the chord and are consistent throughout the parabola. (The parabola graph we use is based on a chord of 1.25 and a perpendicular bisector of 2. See Arched Door problem.)

From the new radius we found on the parabola – x-chord vs. y-radius, we now have to different radius occurring at different positions. Because the change in chord is related to an involute, or unraveling string around a fixed point, (with the exception of its position not being at the center of a circle), we will assume the radius of the encompassing circle that is the max value of the sine curve occurs every Pi radians from the start of the original chord. Now we just have about enough information to solve for the max value of the sine curve.

The most important factor on which all measurements are based is taking the rate of change of the chord to find the angle the new radius will be from the radius of the circle encompassing the max value of the sine curve. This rate was solved by using previous methods of solving for an unknown chord. We need to find out how much the chord changes in Pi radians.

This description is hard to convey in words follow along in the example.

Given: $s=?$, chord = 107, $r= 210 +4$, rate = 1.7801 per π , $\theta=?$

Note this example uses a chord of 107 units.

Since we are using a reference circle of chord 1.25 and radius 2.22512 already found by known equations. We refer to the graph of the parabola and get a radius of 210 +4 or 214.

The next crucial step is finding the rate of change of the circle as the chord is unwinding around its center. An involute increases to its next radius every Pi radians. The rate of change of the radius per turn would be equal to the radius divided by the chord. Remember this rate is the rate of the reference circle and not of the end result. That is because the end result's encompassing circle will be a smaller circle. One in which the circle encompasses both the new chords radius and the max value of the sine curve's radius.

$$rate = \frac{2.22512}{1.25} = 1.7801$$

Now to find the angle the final radius makes unwinding around the circle like an involute.

$$\frac{end\ radius}{rate} * \pi = \frac{214}{1.7801} * \pi = angle\ of\ final\ radius\ in\ radians = 377.676\ radians$$

That angle in radians will be converted to degrees for easier calculations and give a better estimate of how much the angle is.

$$\frac{180}{\pi} = \frac{x}{377.676} = 21639.2 \text{ degrees}$$

$$\frac{21639.2}{360} = 60.109$$

$$\begin{aligned} 60.109 - 60 &= 0.109 \\ 360 * 0.109 &= 39.24 \text{ degrees} \end{aligned}$$

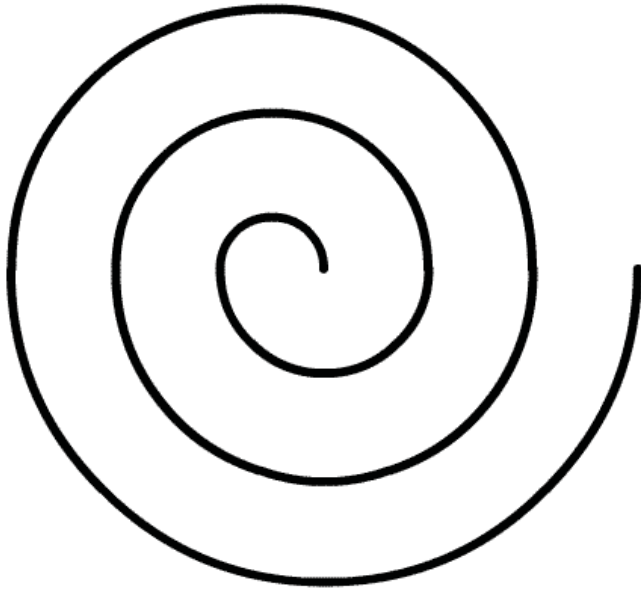
That is to find the angle after wrapping many times around 360 is a percentage of 360 degrees.

$$\frac{\text{chord}}{\sin \text{ of new found angle}} = \text{max value of sine curve} = \frac{107}{\sin(39.24)} = 169.151$$

So an chord of 107 units on a sine curve should be encompassed by a radius of 169.151 which is also the max value of the sin curve.

Attention: It is important to understand this may not work. It is only to outline an idea and create a problem to work on. The one main fault of this work may in fact be the rate of the changing radius. It can be both confusing and challenging to solve. But until these things are found work through the problem and find your own answers.

Constructor's Corner: www.constructorscorner.com
Involute vs. Logarithmic Spiral



Here is an involute with radius's of
 $1.25 * (k * (2.22512 / 1.25))$

Think of an involute as a string revolving around a pole. As the string becomes longer this is what is created.

We can use this to explain curves and to explain circular functions such as the sine and cosine graph.

Read below than study the graphs that follow.

Temporary Work

20060806

As seen a changing circle or rather a changing chord can be expressed by an involute. (See Chord vs Circular Function) We can imagine a string unwinding around the circular function and that will lead to the discovery of all the values on the circular function. The rate of change is the key. The angles that are contained in the involute are proportional. That means that any involute representing the changing rate of the circle can be use.

Pictured here is an involute used to solve a circle starting with a chord of 1.25 units.

Here is the equations:

$2.22512 = \text{radius of first involute arc} = \text{radius of circle with chord } 1.25 = \text{distance covered at Pi radians}$
 $2.22512 * 2 = 4.45024$

$2.22512 / 1.25 = \text{rate of change of involute per arc (at every Pi radians)}$

$1.25 * 2.22512 / 1.25 = 2.22512$

for remaining arcs...

$1.25 * (2 * (2.22512 / 1.25 = 2.22512)) = 4.45024$

$1.25 * (3 * (2.22512 / 1.25 = 2.22512)) = 6.67536$

It is important to note that this involute was drawn by geometric construction. Reference or construction is "Technical Drawing" Tenth Edition, Giesecke page 142

This common to draw an involute of a line was slightly modified to fit the rate of change of $2.22512 / 1.25$. The center of the 2.22512 radius circle was used as the line the involute was constructed around. This is so the unwrapping of the "string" would start at the end of the 1.25 chord. It is difficult to explain in words, but by studying the diagrams it is easily seen.

Recalling the equation $1/(1.25/4) = 3.2...$ Now there is an equation for a logarithmic spiral. If the 1.25 chord was revolved and increased to maintain a circular arc. This logarithmic spiral can be drawn by connected every value of the involute by 2. So maybe we can find a simpler equation for both the involute and spiral.

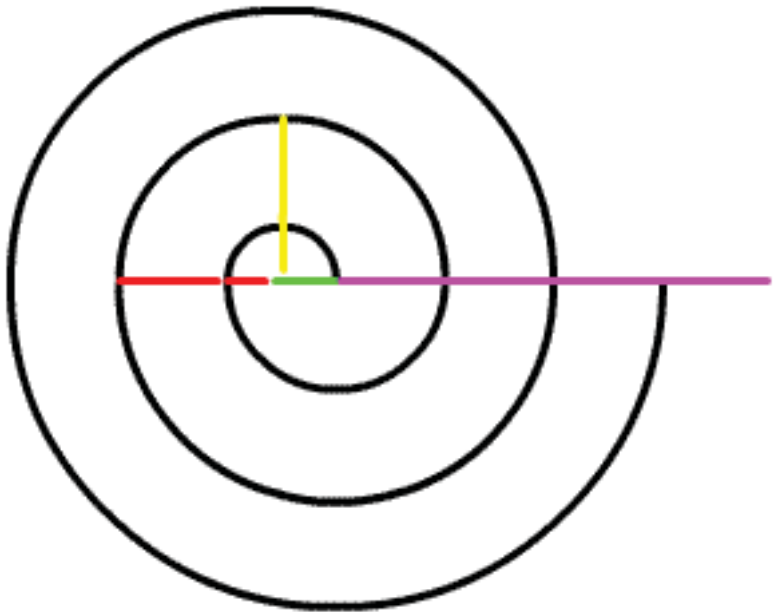
$$3.2 + 1.25 = 4.45$$

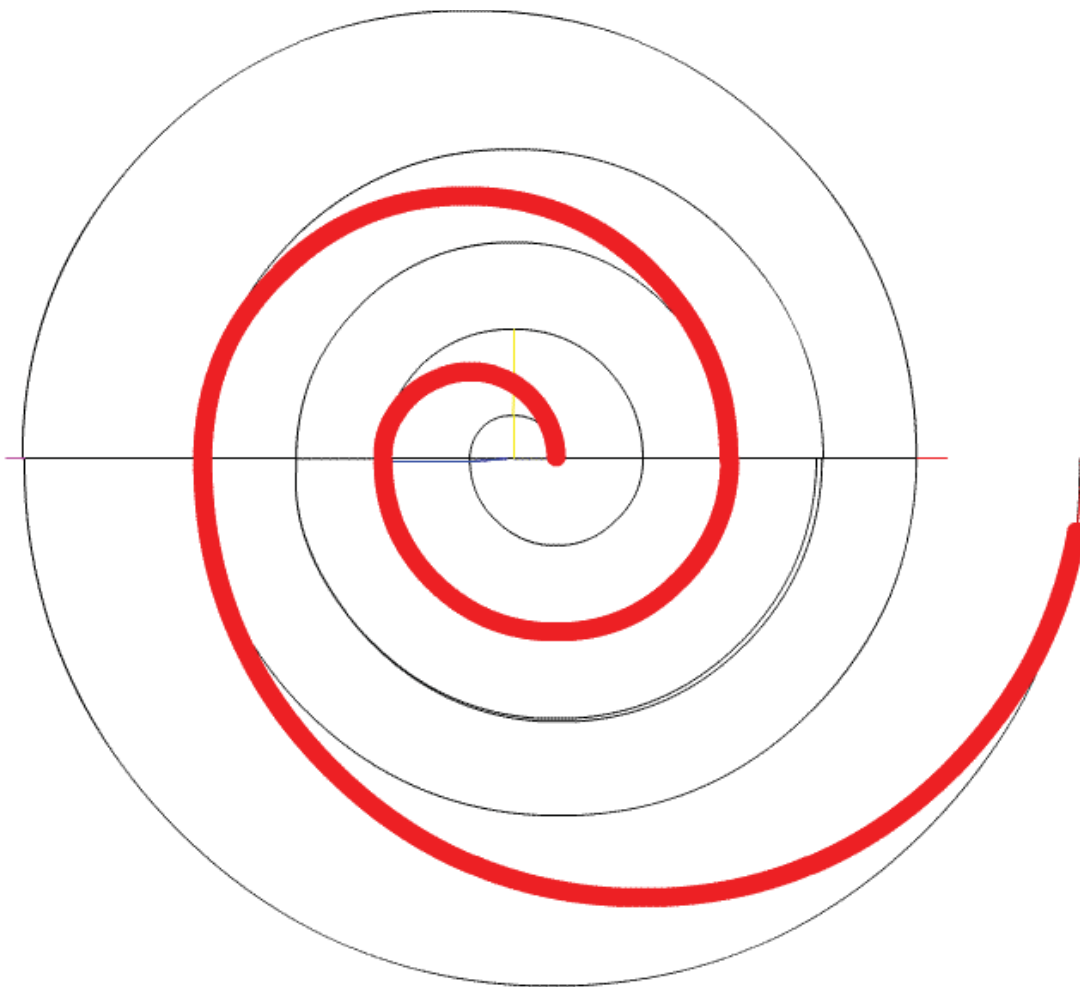
$$4.45/2 = 2.22512 \text{ also the radius}$$

What is the application? Most importantly it describes circular functions such as the sine and cosine. Also it is its own coordinate system. Imagine how curves could be described only using elementary mathematics. But these are yet to be found!

In this graph the small red line plus the length of the horizontal green line at the center is the radius 2.22512 respectively . This involute is based on a change of 2.22512 per Pi radians.

Also pay attention to the two red lines combined. This is the diameter of the circle we solved for. It has a length of 3.2 units. That is taken from the end of the green line. Add the green line and you get the diameter of 4.45 units. If we were to base our shape on the diameter we would have drawn a logarithmic spiral.





This shows the logarithmic spiral drawn over the involute. Recall the equation $1/(1.25/4) = 3.2$.

This is very valuable information when solving for circular function. It would also be a challenge for the curious mathematician to try and find the coordinate system that is created. Also we have found similarities between involutes and logarithmic spirals



*Note do not confuse a logarithmic spiral with the Spiral of Archimedes. Admittedly these spirals can be confusing. A good source for a basic definition is www.wikipedia.org May the Creative Force be with You!

Circle-Parabola-Proportions

Attention: It is important to understand this may not work. It is only to outline an idea and create a problem to work on. The relationship and proportions between slope, parabolas, and circles are interesting. This problem is presented to create interest in those areas. And quite possibly make a little math discovery.

Temporary Work

20060914

So much of the work has been about the relationship of a chord vs. a circle. It has proven to be an interesting idea that can be built upon. From the chord vs. circle to a parabola that describes a circle. To a theory or hunch that explains circular functions such as the sine and cosine.

Einstein said the World and God's Work should be able to be explained in a simple manner that everyone could understand. (This quote is paraphrased.) But in higher mathematics such as calculus sometimes the answer seems to be placed in a puzzle of equations that is difficult to understand the method entirely. But it seems the most useful of knowledge in calculus are those parts that are easily understood. The parts of calculus that are most basic are used as "tools" to solve what is more difficult to calculate. However the important thing to note is that these tools, no matter how logical or basic they may appear, had to be discovered. They are easy to understand, yes, but the discovery of these tools, and any tools we use in life, is a great challenge. A challenge that influences how we see the World. A World that reflects our lives. So to create a math problem is no simple thing. It is just one thing in our lives that we use tools for.

The concept to remember about math is to keep ideas simple. It is no easy task when sometimes math concepts are not tangible. The mathematician has to take the information that doesn't seem to relate and draw comparisons from it. In calculus many solutions to problems can be found graphically. And sometimes still elementary mathematics can do the same job.

Theory Only: Doesn't Work — But the idea is intriguing!

Unfortunately, in this group of theories (which probably don't work) trying to explain a simple concept was taken to a complicated way of doing math. Some calculus was used. But the simplicity comes when apply this knowledge to a problem and not its informal prof.

The Problem:

A tank contains 1000 liters of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?

This problem is solved using calculus refer to mixture problem. Admittedly a relatively difficult problem to fully understand.

But is there another solution? One that would be easier once how to determine the answer once the steps were established. An approach that would show the same problem graphically.

Solution:

20060912

Imagine the rate of change of the salt tank as the rate of change of the circle. That is the circle that is described by the parabola.

The integral is only necessary to describe equations and set up the equation with the rate of change of salt leaving the tank. The goal is to find the equation the equation of the parabola. The known parabola will be modified similar to a way that a sine curve is modified to fit different applications.

20060913

[salt already in tank at time t] - [summation of salt leaving the tank]

$$[0.1 \text{ kg} * 10 * t] - [\int_0^t (\frac{0.1}{1000} * 10 * t) dt]$$

This is the equation of problem. It is also the graph. However it is not in useful form.

The derivative of the equation is taken to find the slope. The slope is the tool we are looking for.

$$f' [0.1 \text{ kg} * 10 * t] = 1 \text{ --- This is a line when graph because the slope equals 1.}$$

$$f' [\int_0^t (\frac{0.1}{1000} * 10 * t) dt] = (\frac{0.1}{1000} * 10 * t) = 0.01 * t \text{ --- This is a parabola when graphed.}$$

The graph of the line minus the graph of the parabola tells how much salt is in the tank at the given time. --- But to put this equation of

$$[\int_0^t (\frac{0.1}{1000} * 10 * t) dt]$$

into a form that can be graphed we must find a way to explain a parabola using the slope. It is similar to finding the slope of a line, however it is difficult to determine how slope effects a parabola.

Using basic algebra we could plot both the line and the parabola from their slope assuming they start at origin (0,0). The relative distance between the line a parabola would be equal to the value we would have if they were subtracted from each other. That is subtract the actual line by the parabola. A new parabola is then formed and its values answer the question of how much salt is in the tank (and how much left) at time t.

20060914

In an attempt to find some relationship between a parabola and its slope, the follow hunch was envisioned. Remember that it is only a hunch and as such is only a idea brainstorm that shows the process of attempts at reaching an answer.

The parabola that describes the salt tank will be compared to the "Parabola Key" parabola we already know the values for.

Here is an example using the "Parabola Key" and the above salt tank problem. It is formed on the basis that a circle (in the form of an involute) is described by a parabola. The fact is that because circles are proportional to each other and a parabola defines a specific circle, this knowledge can be used to relate the slope of the parabola to another parabola. Here is the hunch without testing it in a problem. It is only to show a possibility of little discoveries that are hidden in simple algebra and geometry.

The Equations:

slope of parabola at the value of the circumference of parabola 1
diameter

equals the

slope of parabola at the value of the circumference of parabola 2
diameter

for the Parabola Key ; the known parabola

$$\frac{2(2.22512 * 2\pi) + 8}{4.45024} = 8.080$$

so the proportion = 8.080

$$\frac{0.01 * (t * 2\pi)}{2t}$$

actually the equation is $\frac{0.01 * (\frac{1}{2} t * 2\pi)}{t}$ where t is the first circumference of the drawn by the parabola

However the equation does not give a correct answer. That does not mean that there isn't something important to be noted here. It just means the problem is a hunch. This is presented to augment future work on the problem.

Well since both the parabola and circle have similar, proportional relationship between a circle and parabola of different sizes, can an equation of this relationship be found?

20060917

Since the involute formed by the measurements of the known parabola (The Parabola Key) may form a common proportion that relates to all parabolas, the following equations are formed. The involute increases by a given rate per every Pi radians. This rate will be used to describe the equation of the parabola of the salt tank.

$$\frac{2(\pi * r) + 8}{2 * r} = \frac{2(\pi * 2.22512) + 8}{2 * 2.22512} = 4.9761$$

substituting the number as the proportion:

$$\frac{0.01(\pi * t)}{2 * t} = 4.9761$$

$$t = 6.313363$$

So in the involute that is based on the parabola's measurements we have a radius of 6.3134. This is also the length that the radius of the involute increases per Pi radians, along the involute.

This is an interesting number. However not that this is just a hunch. Much testing and checking of the values needs to be done. This will take quite some time. So instead of not updating the Hunches section, I left an intriguing problem which although has a writeup that is difficult to follow, could lead to more ideas when worked on by more mathematicians. May the Creative Force be with You!

Attention: It is important to understand this may not work. It is only to outline an idea and create a problem to work on. The relationship and proportions between slope, parabolas, and circles are interesting. This problem is presented to create interest in those areas. And quite possibly make a little math discovery.

Parabola vs Exponent

Temporary Work

20061103

Reading about logarithms makes one wonder if logarithms can perform multiplication and division and even simplify exponents. Why does the book say there is no rules for adding? We know this from basic algebra when studying exponents. In other words, $x^2 + y^3 = ?$. Now we realize this is a challenge to solve, but maybe solving the problem completely isn't the goal. Maybe one of the variables x^2 or y^2 can be put into a simpler form of x or y .

The question is, "is it possible to add two different variables that are exponential?" The hunch is yes. Yes that is at least to simplify them by removing one of the variables exponents. In fact we know it is possible. After all, if we know all the variables and exponents we would have no trouble finding the answer.

---20061019---

$$x^3 + y^5 = ?$$

$$4^3 + 7^5 = 16871$$

$$4^7 + [2 * [\text{Log}^{-1}[5 \text{Log}(3)]]] = 16871$$

---20061031---

$$x^2 + y^3 = ?$$

$$4^2 + 7^3 = 359$$

$$4^2 + 7 * [3 * 4^2] + 7 = 359$$

---20061104---

$$x^2 + y^3 = ?$$

$$5^2 + 9^3 = 754$$

$$5^2 + 9 * [3 * 5^2] + 9 * 6 = 754$$

This example is more than using factors. This is using the variables in a different way to calculate the answer. (There is both addition and multiplication here.) So it leads the mathematician to believe there is a series or pattern. The question is what is the pattern with different exponents and variables. (Don't be confused. This example isn't stating this is the correct pattern it just shows a pattern is possible.)

The logarithmic work will be shown first. It is important to note that it is not correct. It is only to present the concept of adding variables and exponents. (20061029) This is no easy problem. It is said to be impossible by many math textbooks. It is "possible" it could be "impossible." However some interesting things can be learned just by following a hunch.

Here is a series of scratch work that will explain the attempts at finding a series among the exponents.

---20061017---

$$x^n + y^k = z$$

$$x^{n+k} + (y-x)^{n+k}$$

$$(y-x)^{n+k}$$

*comment // that is $(y-x)_1 * (y-x)_2 * (y-x)_3 \dots (y-x)_{n+k}$*

$$n+k(\text{Log}(x-y))$$

// after many attempts of finding a logarithmic equation a new approach was tried

20061027

A parabola explains exponents. The key is finding the proportion between 2 numbers on there corresponding parabolas. It is much like adding or subtracting 2 parabolas to form a new one. It would be interesting to see how a quadratic equation is formed from a parabola. That is, the equation of the parabola.

Hunch: Find the values of an exponent by the summation of the values of an involute. (See Chord vs. Circular Function)

20061028

It would be interesting to see just what happens with the summation of the values under the graph of the parabola of the exponential function. If the exponent is know than so is the value of the parabola. For example the graphs of $y = x^2$ or $y = x^3$. Maybe there is a pattern.

---20061028---

$$4^2 - 2 = 1^2 + 2^2 + 3^2 = 14$$

$$5^2 + 5 = 1^2 + 2^2 + 3^2 + 4^2 = 30$$

$$4^2 = 30 - 14 = 16$$

$$6^2 + 19 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$$

$$5^2 = 55 - 30 = 25$$

$$\dots 6^2 = 91 - 55 = 36 \dots 7^2 = 140 - 91 = 49$$

similarly

$$5^3 - 25 = 1^3 + 2^3 + 3^3 + 4^3 = 100$$

$$6^3 + 9 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$5^3 = 225 - 100 = 125$$

$$7^3 + 98 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = 441$$

$$6^3 = 441 - 225 = 216$$

// So there is a pattern. Just common sense and not hard to see. It is subtracting an earlier value from the next higher value. Simple as it is, it may prove useful in the task of adding and subtracting numbers of different exponents.

So far solution attempts look amateurish...like the steps are not following the fundamental rules. (This is true of the logarithmic equations which are not completely shown.) However on further inspection nothing could be further from the truth.

Finding an quadratic equation from a parabola or using 2 parabolas (of certain exponential functions) to add $x^2 + y^3$ is possible.

Looking "amateurish" is ok at first. These are new ideas. It is meant to be a learning process. The problem has to start somewhere.

---20061028---

$$[\int_1^{n_1+1} n_1+1 \, dn] - [\int_1^{n_1} n_1^2 \, dn] = n_1^2$$

$$\text{for } x^2 + y^3$$

$$\{[\int_1^{n_1+1} (n_1+1)^2 \, dn] - [\int_1^{n_1} (n_1)^2 \, dn]\} + \{[\int_1^{n_2+1} (n_2+1)^3 \, dn] - [\int_1^{n_2} (n_2)^3 \, dn]\}$$

$$//\text{which equals } n_1^2 =$$

$$//\text{which equals } n_2^3$$

$$\text{let } u = (n_1 + 1)$$

$$\int_1^{n_1+1} u \, du = \left(\frac{u}{3}\right)^3 + c$$

$$= \left[\frac{n_1+1}{3}\right]^3 + c$$

//As easily seen this is becoming much to complicated. The original $x^2 + y^3$ was easier. However it is just a step in finding what direction the problem solving efforts should follow.

20061030

The next logical step after trying the integral is to use the rate of change or as it is known in calculus, the derivative. (20061104)— The derivative or rate of change is the key to putting x (in x^2+y^3) in terms of y so that they can be added or subtracted.

20061104

The goal of adding x^2+y^3 is eliminating at least one of the variables exponent. We can do this by finding out how to add the smaller number to the larger with the smaller number value's size described the same way as the larger value. In other words, the goal is to find how much x^2 is so much of y^3 . Admittedly it is confusing, but hopefully the examples will show what is being attempted. And it will be shown how the derivative (rate of change) helps accomplish this.

Show equation attempts — Note these attempts are not ready to share at this time.

Note: At this time this is where the work stops. It is usually taught in class and textbooks that there is no rule for adding exponents. And it is considered an unsolved problem that may have no result that would be useful when solving equations. This is just an attempt to show how simple and fun it can be to work on an idea or concept. Approaching an unsolved problem is a challenging task, understated, but the things learned lead to other ideas. And it is a good warmup for other problems. It just might lead to great brainstorming sessions.

20061105

It is important to note these examples and equations do not work for numbers greater than 10. (That is the first examples that were presented.) However, this should not discourage work! That is because this hunch is based on a parabola. The graphic representation of this problem is clear. It is summing the places of the exponential graph (a parabola) to form a new graph representing x^n+y^k . This is much like adding or manipulating the graph of a sine curve.

Also note previously unsolved math has been posted on Constructor's Corner. Sometimes it is unsolved. But some problems have been solved because the problem and data were presented in a way that helped organize problem solving efforts. So this information is placed here not as an answer but as a way to organize problem solving efforts.

This is where the work stops on this document for now. I will continue to tinker with ideas. But this impossible problem is just one of many that can be worked on together on Constructor's Corner. 8 people voted on a poll on Constructor's Corner and expressed interest on working cooperatively on a math project. This is just a few to get started. 8 people are more than enough to solve problems and find some great answers. A Wiki where people can edit web pages and add content is on the way... But until then, May the Creative Force be with You

Video Game Curves

Temporary Work

20061219

Since November 2006 the World was introduced to the Playstation 3 and the Nintendo Wii. Both have motion sensing chips to detect motion. The Wii also has a motion detecting infrared camera. The question is how will a motion controller change game play. Is it simply just moving the controller or will it add to the game experience?

The X-box 360 has been out since November 2005. The controller was redesigned, but it did not include motion sensing. Admittedly the controller has improved and is a superior controller for shooter games. The controller is also compatible with the PC. However the Xbox 360 does have some technology that just might be better than a motion sensing chip. Instead the Xbox uses a camera with motion sensing software. It started with a racing game controlled by hand movements. But this is only the start. There have been game concepts proposed such as casting a spell with hand movements or a sword battle game. This is one technology Microsoft will have to develop if the new game play requires motion sensing technologies.

The camera is said to have limited technology, but that might just be wrong. This Hunch will present a way to describe unit using mostly elementary mathematics such as geometry and trigonometry. It will show how simple math can create a game with 3D sword play. This is not just swinging a controller in a circle to spin the character. Think of a green sword (colored like a movie effect's blue screen.) The motion of this sword and the players arm are processed by the camera as the player fights a 3D sword's man onscreen. This movement can be described simply by math so that it can be programmed into the Xbox.

20061123-20061221

Ok, how does recording 3D sword motion by the game system (Xbox 360) happen? Well motion captor has been established. Often actors or athletes were "ball shaped" markers at the joints that cause the movement of the body. So the web cam of the Xbox 360 has built in motion detection. (Note I do not know the limits of the motion sensor or its ability to recognize objects.) So it is assumed that if a player was to were "ball shaped" markers the camera would be able to sense the motions of the joints and measure the distance between joints. Also it would be helpful if the camera could measure the relative size of those objects on the screen. For example if a marker moved closer or further away from the screen.

Theory—

If we have a sword and 4 markers on the joint of the arm: the 3D motion of a sword can be read by a motion sensing camera. Also this movement will be create the true 3D movement of the sword in the video game.

This theory is based mathematically on the relative velocity problems common in the study of dynamics. Also the sine curve from trigonometry will be used to simplify what would otherwise be very rigorous and complicated math. This problem will set each joint of position of sword as an individual sine curve. These curves will be compared to what is similarly done with alternating current in electricity. In other words, the angle which the sine curves are out of phase will help determine the position of the sword.

Key Ideas to Review First

1. The members, the arm's joints and top and bottom of sword, are a set of 2D lines which when moved together create a unique and special curve.
2. The foreshortening of the arms and sword give the 3D position.
3. There is a sine curve for every member. This sine curve is compared to the other members as if it were "out of phase."
4. If the phase angle of any member is changed: it is no longer the same curve. That is the position of the swing of the sword has changed it's curve.
5. There is a need to account for a full swing. That is the swing of the sword from right to left. Note however, not only does this increase or decrease the flat 2D picture of the members in the camera it places them on the opposite side of the body. But it must also be noted the members are still revolving in a 2D circular path in the viewpoint of the camera.
6. A 2D sin curve could only happen with the given measurements (of the given members.) In other words if the 2D result of the members is less then their start measurements that length would have to be accounted for in the 3D plane (Z-axis.) — So each curve, even though seen by the camera in 2D, is unique and it simplifies calculations. It allows the entire technique to work.
7. This technique has many other applications. Combined with the parabola vs sine curve, it can be used to solve dynamics problems such as relative velocity problems. The key is a proportion between velocity and position of the members using sine curves and phase angle. Sounds complicated but it is not. It just needs explored further.

20061224

Arm + Sword = fixed length

So in theory measurements and position of the swing could be found using the sine curve and phase angle!

Hopefully it is clear what is being attempted to be solved here. I will post updates to better explain and hopefully solve this problem. This is a good group project. If you have read this and want to work on a problem email: trurlthe_constructor@hotmail.com . Also more math can be found in the math_hunches section of Constructor's Corner.

Sonar Coordinates

Temporary Work

20060102

Often when exploring a new math problem it is beneficial to start out with the theory and then prove it. Not only is this a good place to start, but it gives the reader an understanding of what the math is intended to mean. Often when creating a new math application you start with the theory and build from there testing the hypothesis.

The field of dynamics deals with movement, velocities, and force. As shown in the `video_game_curves` problem, when objects move in straight lines we can use a sine curve to figure out angles, velocities, distance traveled, and force. We start with creating a sine curve for each member (member meaning each length of straight line). We are interested in determining how one straight line effects the next straight line and so forth until the movement has been described. Then we graph the lengths with it max and min values as if it would revolve in a circle. Once a sine curve has been established for all lengths, the sine curve is then summed to give one sine curve. It is also relevant to find the phase angles between these sine curves.

The part of the theory that needs to be verified is how exactly the length of the lines in rotation effects the over all velocity. And since we know velocity we can find force needed to move a certain distance. See `Chord_vs_Circular_Function`. This theory is based upon relative velocity (a member such as the given force + its length as it rotates around that given force). It also helps graph and maybe even put into a useable form for the computer a complex relationship between members. That is the result from the many vectors of the members.

So if we know the angular velocity or instantaneous velocity, there is the question, "What is the relationship between a member as it rotates around the reference member + the velocity of the reference member, both, compared to second members position with the combined members?" In other words if the sine curve of the members are known and the instantaneous velocity at a given point is found, "Does the value of the new found instantaneous velocity relate proportionally to the sine curve of members length as they rotate around a circular path?" Simply put is there a relationship between velocity and members position? If there is can we use `Chord_vs_Circular_Function` to find the unknown sine curve values?

Applications?

20060101—20060102

Imagine a pool game going on, on a pool table. Only one chance to make the perfect shot. If we measure the angles and sum the phases of the sine curve, we find where the ball will travel. But in order to make the shot the ball must move at the correct velocity. Since $\text{force} = \text{mass} * \text{acceleration}$ we know the force need to make the shot. So how fast the stick moves combined with the proper angle make the shot. It would be complicated to measure the speed of a pool stick hitting the cue ball. Lasers, gears, and stop watches aren't need. A simple device that plays a tune to the time (duration) to the time of speed that the stick should move to hit the ball. Not only would this give the user a rhythm and timing (It would be like playing an instrument.), the user would learn the proper technique.

** It is also important to not that the integral of the balls combined, member sine curves equals the total distance the ball travels.

Also it might be interesting to ask, "Is there a coordinate system here?" Specialized coordinate system can be fabricated to fit the needs of the individual math problem. This is nothing new. Imagine a rectangle (pool table), triangle, or any other measurable shape with balls bouncing inside the shape. After many collisions or reflections inside the shape, the path of the ball is changed accordingly. Angles return and give the angles of the shape. That is where the name sonar_coordinates comes from. Just a "hunch."

Hopefully it is clear what is being attempted to be solved here. I will post updates to better explain and hopefully solve this problem. This is a good group project. If you have read this and want to work on a problem email: trurlthe_constructor@hotmail.com . Also more math can be found in the math_hunches section of Constructor's Corner.

Constructor's Corner: www.constructorscorner.com

Arch Doorway Teacher's Solution

I have had a solution to the arc doorway problem on my website for quite some time. It basically solves the value of a circle given a chord (or change of cosine or sine). I have left the solution without any updates. Updates meaning additional solutions or new applications of the solution. I have tried to use simpler methods to determine the size of the circle formed when only its cord is given, but am still tinkering with the equations. One solution to the problem, I never posted, but it is a valid solution using higher mathematics of calculus. The solution was given by my trigonometry instructor (A. Whitcomb) during Point Park College Spring 1998 semester (Now Point Park University). The solution utilizes Newton's Method. It uses the Taylor Series of the cosine for its estimation of $f(x)$. (I admit to not fully understanding all parts of the solution, but know enough to follow along and try to find any way it relates to my solution of the problem.)

Here it is scanned in the same form that was given to the class members. Maybe there is a way to relate the two solutions or at least find something that relates between the solutions. Or maybe a new approach to curves or circles will be found.

Problem 71 on Page 281

Problem 71, where on the arch part you are given the length of the chord and length of the arc of a circle, can not be solved directly.

If we consider the arc as part of a circle of radius r and central angle θ , we have the two equations

$$(1) \quad r \theta = L = \text{length of arc and } 2 r \sin \theta/2 = C = \text{length of chord}$$

If we look at the ratio $L/C = r \theta / 2 r \sin \theta/2 = 1 / \{ \sin \theta/2 / \theta/2 \}$ and use the fact that the $\sin \theta/\theta$ is a decreasing function for $0 < \theta < \pi/2$, we see that we have a solution for the equations in (1) if and only if $1 < L/C < \pi/2$, and in that case the solution is unique.

Since in the case we are looking at $L/C = 5/4 = 1.25$, the configuration with the numbers given there is possible and there is a unique solution for r and θ which determine the area of the arch portion of the door. We have

$$(2) \quad \begin{aligned} r \theta &= L = 5 \\ C^2 &= r^2 + r^2 - 2 r^2 \cos \theta = 16 \end{aligned}$$

So to determine the area of the arch portion of the door, we must solve the equations in (2) and use the formula for the area of a segment given in Problem 69. These equations can be written as

$$r \theta = 5 \quad \text{and} \quad r^2 (1 - \cos \theta) = 8$$

and squaring the first equation and substituting in the second equation, we have for θ that

$$(3) \quad \cos \theta = 1 - \frac{8}{25} \theta^2$$

This equation is not solvable by the techniques in Section 5.4 and numerical methods like Newton's method that you may have seen in a course in calculus must be used. We proceed using this technique to solve Problem 71.

We first estimate the solution by replacing $\cos \theta$ by $1 - \theta^2/2 + \theta^4/24$, an estimate for it using the first terms of its Taylor's series, and solve the resulting equation for θ .

$$1 - \theta^2/2 + \theta^4/24 = 1 - 8/25 \theta^2$$

or $\theta \approx 2.078461$. With this initial estimate we now use Newton's method to approximate the solution for θ in (3). We use the formula

$$\theta_{\text{new}} = \theta_{\text{old}} - f(\theta_{\text{old}})/f'(\theta_{\text{old}})$$

to generate successive finer approximation to the value of θ . We rewrote (3) in the form $f(\theta) = \theta^2 - 25/8 (1 - \cos \theta) = 0$ to apply Newton's formula.

i^{th} step	θ_{old}	$f(\theta_{\text{old}})$	$f'(\theta_{\text{old}})$	$\theta_{\text{new}} = \theta_{\text{old}} - f(\theta_{\text{old}})/f'(\theta_{\text{old}})$
1	2.078461	-0.324180	1.426040	2.305790
2	2.305790	0.096097	2.293346	2.263887
3	2.263887	0.003566	2.123790	2.262208
4	2.262208	0.000006	2.117083	2.262205
5	2.262205	0.000000(-)		

Using 2.262205 for θ , we then have $r = 5/2.262205 = 2.210233$ ft.

The area of the door is now given by the $4(8) + \frac{1}{2} (2.210233)^2 (2.262205 - \sin 2.262205) = 35.643955 \text{ ft}^2 \approx 35.64 \text{ ft}^2$.

000

Baby Seat - Dynamics

MET/CET 101

Quiz 2

Name: _____

The Hsu's bought this seat to accommodate 40-lb Avery at their dining table.

Draw a FBD of seat, with Avery in place.

Calculate the forces exerted on the table.


Be sure to answer why rubber tips must be securely attached. ^{what would happen without the tips?}

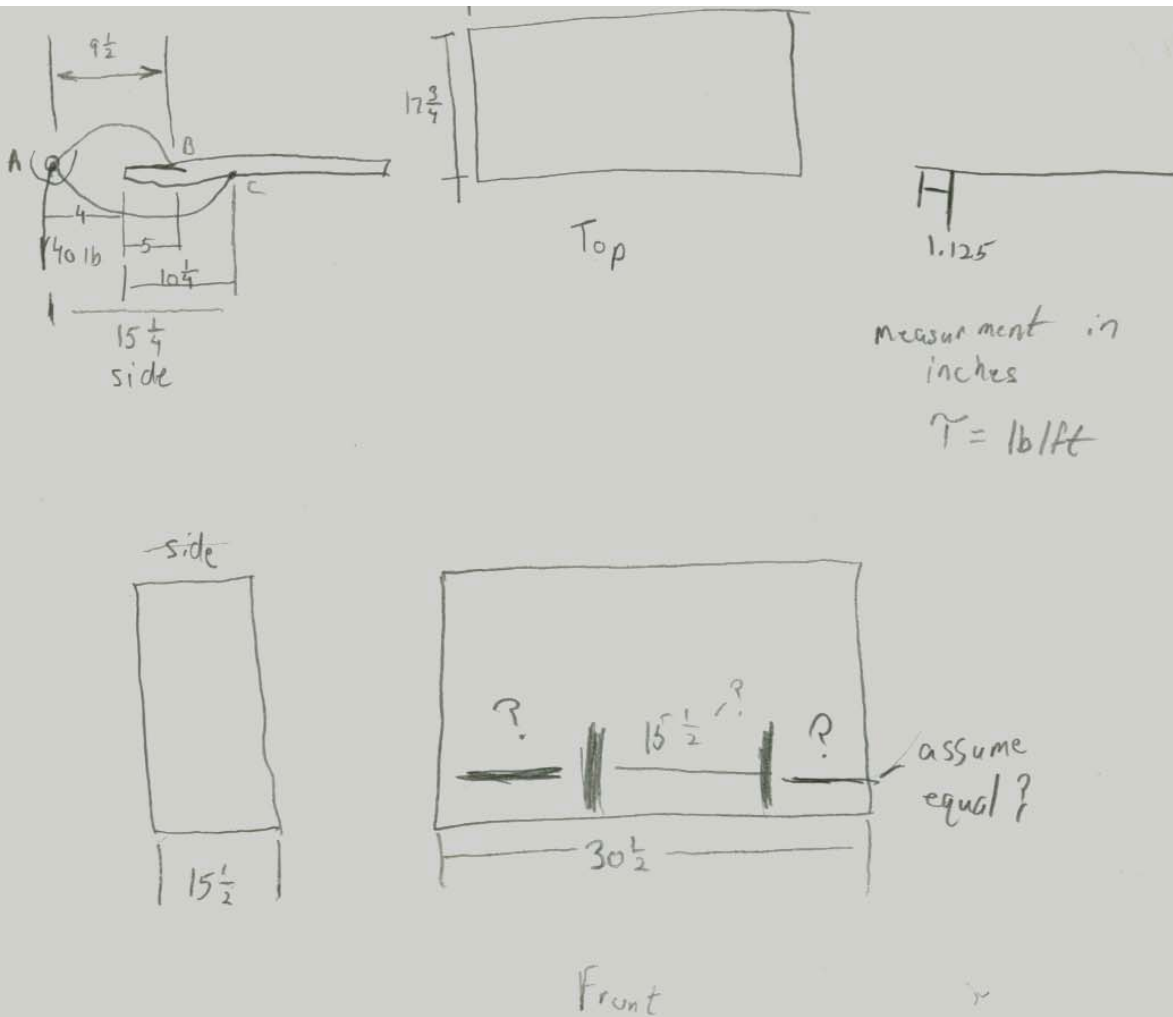
Submit team report (2 pg typed with diagrams and calcs by Nov. 5 1998).

Bonus

Find the weight of the heaviest child the chair will support without tipping the table.

Have Fun!

 I don't think it will ever tip a table if it sits on securely because of the way it pushes on ^{and it supports each other.} the table. The forces are vertical. The seat will never angle enough so there can be a moment acting on the table. Even if it did the pivot point would be between the forces. If the upward force does overcome the weight of table the downward force would be prevented by the table legs.



cantilever

The table will bust before tipping. That is why cantilevers are the strongest type of supports. They prevent movement in all directions. Any thing which is going to support a child has to be safe. Any weight limit on the baby is probably due to the friction force of the rubber stoppers from any movement of the baby.

The following is a group project I participated in with 5 other class members. It was the second part of a quiz. We were allowed to take this second part of the test home. The only thing that was know was the weight of the child and the measurements which we, the students, recorded as being important to solve for the forces of the seat. No cheating by lifting the seat was allowed.

This original problem was presented in a statics course by Associate Professor: Alan D. Chamberlain, PE --- Point Park College (Now Point Park University)

Here is the problem:

The Hsu's bought this seat to accommodate 40 lb Avery at their dining table. Draw a FBD of seat, with Avery in place.

Calculate the forces exerted on the table. Be sure to answer why rubber tips must be securely attached.

Submit team report (2 pg typed with diagrams and calcs by Nov. 5 1998.

Bonus

Find the weight of the heaviest child the chair will support without tipping the table.

Have Fun!

Here is the solution that was turned in. Notice any errors?

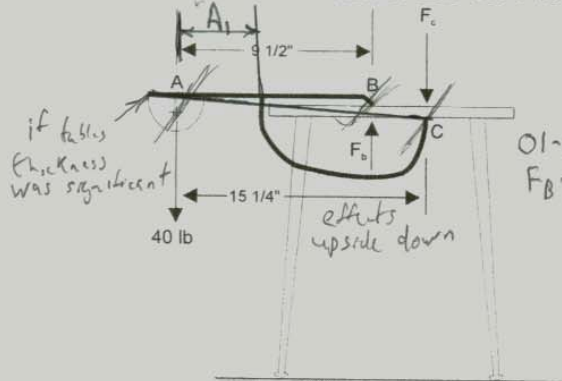
Given: "Baby-sitter" apparatus with 40 lb. baby secured, school table, 16' tape measure and six brains of varying activity.

Find: A) Forces exerted on table while 40 lb. baby is seated,
B) Weight of heaviest child table can hold (assuming table is 30 lb.)

**MET 101 A
STATICS
QUIZ 2**

**CHARLES HURKA
PAUL KRUPZIG
NICK KUCEL
AMBER RUSSELL
CHRISTIAN SAUER
ROBERT SNYDER**

**PART A
FORCES EXERTED ON TABLE**



01-12-07
 $F_B = 9.5 \cdot F_A$

$$F_b + F_c = 0$$

$$\begin{aligned} \Sigma M_a = 0 \\ +40(9 \frac{1}{2}) - F_c(5 \frac{3}{4}) = 0 \\ 66 \frac{2}{23} \text{ Lb} = F_c \\ \text{divide by 2 for each leg} \\ F_c = 33.04 \text{ Lb} \end{aligned}$$

$$\begin{aligned} \Sigma M_c = 0 \\ +40(15 \frac{1}{4}) - F_b(5 \frac{3}{4}) = 0 \\ 106 \frac{2}{23} \text{ Lb/in} = F_b \\ \text{divide by 2 for each leg} \\ F_b = 53.04 \text{ Lb} \end{aligned}$$

$$\begin{aligned} \text{check answer} \\ \Sigma F = 0 \\ -40 + 106.09 - 66.09 = 0 \\ \text{check!} \end{aligned}$$

THE RUBBER GRIPS ON THE LEGS
PROHIBIT HORIZONTAL MOVEMENT
(AND PROTECT FURNITURE)

Did Nick lack these lever up?
 $40(9.5) - F_c(15.250)$
 $F_c = 24.918 \text{ lb}$

$40(15.250) - F_b(9.5)$
 64.2105

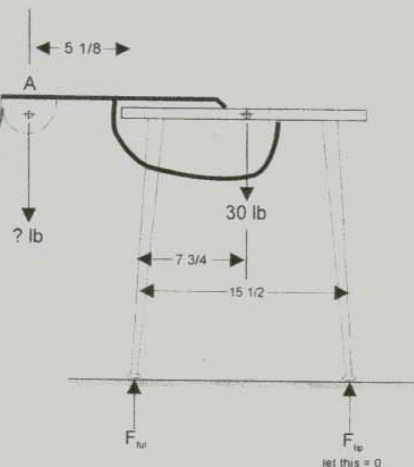
01-12-07
 $F_B = [F_A \cdot \text{lever } A_1]$

$$\begin{aligned} F_B &= [F_A \cdot 9.5] - [F_{A_1} \cdot F_B] \\ F_c &= [F_B \cdot A_1] + [15.250 - A_1] \end{aligned}$$

**PART B
HEAVIEST CHILD THIS TABLE CAN HOLD**

01-13-07

F_c is longer
to increase upward
force and hold
up F_B



$$\Sigma M_{p_a} = 0$$

$$+[(? \text{ Lb.})(5 \frac{1}{8})] - [(30)(7 \frac{3}{4})] - [(0)(15 \frac{1}{2})] = 0$$

$$? = 45.37 \text{ lb}$$

This is when $F_{up} = 0$
(That's when it will tip)

$$? = 45.37 \text{ lb}$$

HEAVIEST CHILD THIS TABLE CAN BEAR

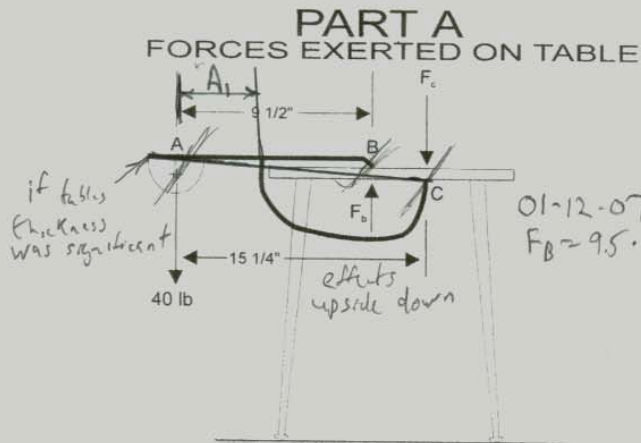
**WARNING: COPYRIGHT ©1998 HURKA,
KRUPZIG, KUCEL, RUSSELL,
SAUER AND SNYDER.**

Given: "Baby-sitter" apparatus with 40 lb. baby secured, school table, 16' tape measure and six brains of varying activity.

Find: A) Forces exerted on table while 40 lb. baby is seated,
B) Weight of heaviest child table can hold (assuming table is 30 lb.)

**MET 101 A
STATICS
QUIZ 2**

**CHARLES HURKA
PAUL KRUPZIG
NICK KUCEL
AMBER RUSSELL
CHRISTIAN SAUER
ROBERT SNYDER**



$$F_b + F_c = 0$$

$$\begin{aligned} \Sigma M_c = 0 \\ +40(9 \frac{1}{2}) - F_a(5 \frac{3}{4}) = 0 \\ 66 \frac{2}{23} \text{ Lb} = F_c \\ \text{divide by 2 for each leg} \\ F_c = 33.04 \text{ Lb} \end{aligned}$$

$$\begin{aligned} \Sigma M_b = 0 \\ +40(15 \frac{1}{4}) - F_c(5 \frac{3}{4}) = 0 \\ 106 \frac{2}{23} \text{ Lb/in} = F_b \\ \text{divide by 2 for each leg} \\ F_b = 53.04 \text{ Lb} \end{aligned}$$

$$\begin{aligned} \text{check answer} \\ \Sigma F = 0 \\ -40 + 106.09 - 66.09 = 0 \\ \text{check!} \end{aligned}$$

THE RUBBER GRIPS ON THE LEGS
PROHIBIT HORIZONTAL MOVEMENT
(AND PROTECT FURNITURE)

Did Nick lack these lever up?

$$40(9.5) - F_c(15.250)$$

$$F_c = 24.918 \text{ lb}$$

$$40(15.250) - F_b(9.5)$$

$$64.2105$$

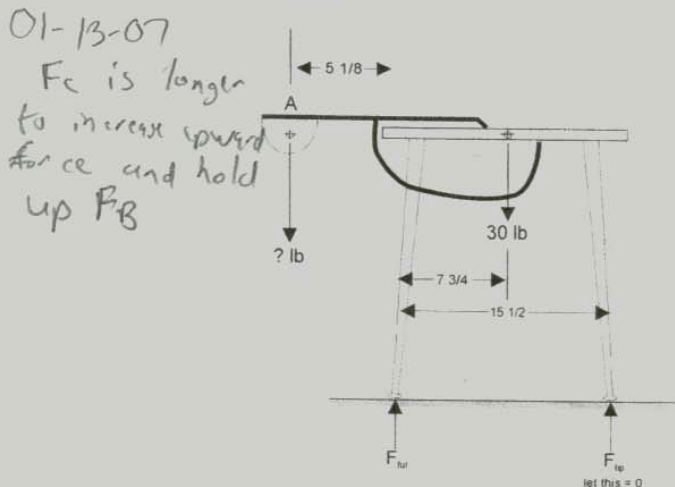
01-12-07

$$F_B = [F_A \cdot \text{lever } A_1]$$

$$F_B = [F_A \cdot 9.5] - [F_{A_1} \cdot F_B]$$

$$F_c = [F_B \cdot A_1] + [15.250 - A_1]$$

**PART B
HEAVIEST CHILD THIS TABLE CAN HOLD**



$$\begin{aligned} \Sigma M_{p_0} = 0 \\ +[(? \text{ Lb.})(5 \frac{1}{8})] - [(30)(7 \frac{3}{4})] - [(0)(15 \frac{1}{2})] = 0 \\ ? = 45.37 \text{ lb} \\ \text{This is when } F_{sp} = 0 \\ \text{(That's when it will tip)} \\ ? = 45.37 \text{ lb} \\ \text{HEAVIEST CHILD THIS TABLE CAN BEAR} \end{aligned}$$

WARNING: COPYRIGHT ©1998 HURKA, KRUPZIG, KUCEL, RUSSELL, SAUER AND SNYDER.

20070116

As one, astute mathematician, may realize from the group project is that the common and simple mistake of using the wrong units was made. The length of the baby seat arms (which act as levers) were multiplied by the pounds of force. This gives pounds of force per inch. What we want is pounds of force per foot so the lengths must be divided by 12 because there is 12 inches in 1 foot. This is the more common English unit for torque: pounds per foot also commonly referred to as foot pounds.

My solution: (It is important to note it may still be incorrect.)

This problem is taken after learning about forces on trusses. However the way the baby-seat is positioned it is held in place by the supports acting as levers. So both the levers and forces on the table must be accounted for.

The Math:

F_B is held in place by the upward force of F_C --- As at the same time F_C is kept from falling by the downward force of F_B

It is important to not F_B and F_C work as a team. The force of the child's weight would lift point F_B of the table if F_C did not put a downward force on F_B . The same force at F_B is what holds F_C from falling due to gravity.

$$\begin{aligned} F_C &= F_A * 15.250 \\ &= 40lb * 15.250 \\ &= 610lb/in \end{aligned}$$

$$610/12 = 50.85$$

$$F_C = 50.85 \text{ lb/ft}$$

Now find F_B

$$\begin{aligned} F_B &= F_C - [F_A * (9.5/12)ft] \\ &= 19.16 \text{ lb/ft} \end{aligned}$$

As for finding the maximum force (weight) that the baby seat can hold. I believe in theory the weight is practically limitless. It depends on the strength of the table legs and construction of the baby seat. But for this particular table it could flip the table by acting on the tables center of gravity.

Also it may be relavent to note the proportion between F_B and F_C .

$$19.16/50.85 = 0.37679$$

$$F_B = 37.679\% * F_C$$

I have yet to accurately determine the maximum wait the table and seat can hold. It will be left for future projects

References:

Schaum's Outlines: Engineering Mechanics - Statics and Dynamics, fifth edition, Chapter 5, to review trusses

Wikipedia.org, http://en.wikipedia.org/wiki/Torque#Conversion_to_other_units to check units of torque

